External Shocks and FX Interventions in Emerging Economies*

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This paper discusses the role of sterilized foreign exchange (FX) interventions as a monetary policy instrument for emerging market economies in response to external shocks originated from three correlated fundamentals: global GDP, foreign interest rate and commodity price movements. We develop a model for a commodity exporting small open economy to analyze the implications of FX interventions. We consider FX interventions as a balance sheet policy induced by a financial friction in the form of an agency problem between banks and its creditors (domestic and foreign). The severity of banks' agency problem depends directly on a measure of currency mismatch at the bank level. Moreover, credit and deposit dollarization coexist in equilibrium as endogenous variables and the UIP condition does not hold. In this context, FX interventions can lean against the response of banks' lending capacity, and ultimately the response of real variables, by moderating the exchange rate response via two mutually reinforcing effects: exchange rate stabilization and lending capacity crowding out induced by the sterilization process associated to it. Furthermore, we take the model to the data by using limited information approach based on an impulse response matching function estimator. Our quantitative results indicate that, conditional to external shocks, FX interventions can successfully reduce output and investment volatility, and generate meaningful welfare gains when we compare it to a free-floating exchange rate regime. Instead, when banks' agency problem depends on an industry measure of currency mismatch, banks do not internalize the effects of borrowing and lending in foreign currency on the severity of the agency problem and the UIP equation holds with equality. In this case, even though the incentive constraint binds, FX interventions are irrelevant for the aggregate equilibrium.

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Emerging market economies (EMEs) face volatile external shocks that have shaped capital flows and exchange rate dynamics since the collapse of the Bretton Woods system and more recently due to global financial integration. For instance, three relatively recent global events had significant implications for EMEs: the global commodity boom originated by China's

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strong demand during the 2000s, the expansionary monetary policies in major advanced economies in response to the Global Financial Crisis, and the normalization of the Fed's accommodative monetary policy (also known as the Taper Tantrum). These external shocks have different fundamentals, which can be summarized in terms of three main interrelated components: global demand, foreign interest rates, and commodity prices. Capital flows to EMES affect domestic financial conditions and credit growth through the availability of foreign currency denominated funds and exchange rate fluctuations, which in some cases have placed the financial system in a more fragile situation.

Many central banks, especially in EMEs, responded to these events by building FX reserves during capital inflow episodes. These central banks were considered to be in a good position to deal with capital reversals and effectively sold those accumulated reserves during capital outflow episodes. Specifically, EMEs have relied on sterilized FX interventions (i.e., official FX purchases or sales aimed at leaving domestic liquidity unaffected) to smooth out the impact of rapidly shifting capital flows and reduce exchange rate volatility while providing businesses and households with insurance against exchange rate risks. Moreover, foreign currency debt in EMEs has been increasing, leaving them more exposed to global financial flows; and therefore financial stability has become an important objective of FX interventions.[1] The mix of policy tools used by policy makers in EMEs also includes macro-prudential measures and capital controls[2]. The effectiveness of these tools is still under debate and more research is needed to make a better assessment of these instruments as a complement to conventional interest rate policy.

The purpose of this paper is to develop a macroeconomic model to analyze FX interventions as a monetary policy tool that takes on attributes of a financial stability instrument as a response to external shocks. We define FX interventions, as a situation, where the central bank buys/sells FX with the banking system in exchange for domestic currency-denominated assets but in a way that offsets any change in the supply of domestic liquidity by changing the amount of domestic bonds, issued by the central bank, in hands of the banking system. In line with Chang (2019), we view FX intervention as a non-conventional monetary tool induced by the existence of financial frictions in the domestic banking sector. In particular, when the relevant financial friction binds, leverage constraints restrict banks' balance sheet capacity and limits to arbitrage emerge together with widening interest rate spreads. Only in the financially constrained equilibrium, FX interventions affect the equilibrium real allocation, since it relaxes or tightens the financial constraint that banks face.

In our framework, FX interventions affect the economy via two mutually reinforcing effects: exchange rate stabilization and lending capacity crowding out induced by the sterilization process implemented with an FX intervention (similar to the empirical findings of Hofmann et al. (2019).[3] We suggest, however, that the financial friction approach to FX interventions differs from unconventional monetary policy for closed economies in several aspects. The unconventional monetary policy literature emphasizes that the conventional instrument is active until the policy rate reaches the effective lower bound. Only in those cases, central banks might deploy balance sheet policies such as QE, LSAP, or credit policies. On the contrary, we consider that financial constraints are binding in EMEs even in "normal" times. Moreover, we argue that for EME inflation targeters, FX intervention might be considered a

[1] The existing literature have identified four main policy objectives for using FX interventions: financial stability, price stability, precautionary savings (after experiencing crisis in the 80-90s), and export competitiveness, In this paper, we focus in the first two. See Arlans and Cantú 2019, Patel and Cavallino 2019, Chamon and Magud 2019, Hendrick et al 2019, and Chamon et al 2019.

[2] See Céspedes et al. (2014) for a discussion of recent LATAM central banks' experiences.

[3] See Céspedes et al. (2017), Chang (2019), and Céspedes and Chang (2019) for similar frameworks that introduce FX interventions as an unconventional policy tool.

balance sheet policy that is active in normal times, as well as during credit crunch or sudden stop episodes. Contrary to Chang (2019), we suggest that what really matters in EMEs is how tight financial constraints are and not necessarily if those constraints bind or not.

We build a general equilibrium model for a commodity exporting small open economy where FX interventions are relevant for the equilibrium allocation. In our framework, the central bank follows a Taylor rule to set its monetary policy rate (conventional monetary policy) but also "leans against the wind" in response to exchange rate fluctuations. The model is an extension of Aoki et al. (2018) (henceforth ABK) where banks face an agency problem that constrains their ability to obtain funds from domestic households and international financial markets. Like in Gertler and Kiyotaki (2010), Gertler and Karadi (2011), Gertler et al. (2012), and Gertler and Karadi (2013), the agency problem introduces an endogenous leverage constraint that relates credit flows to banks' net worth and ultimately makes the balance sheet of the banking sector a critical determinant of the cost of credit faced by borrowers. In this context, unconventional monetary policies or balance sheet policies have real effects.

Our model departs from ABK in three key aspects. First, the banking system is partially dollarized on both sides of its balance sheet and exposed to potential currency mismatches and sudden exchange rate depreciations as it is the case in many EMEs that show a high degree of vulnerability to external shocks. Therefore, credit and deposit dollarization coexist in equilibrium as endogenous variables. On one hand, we assume that intermediate good producers must borrow in advanced from banks in order to acquire capital for production but needs a combination of domestic currency and foreign currency denominated loans to buy capital. The combination of both types of loans is achieved assuming a Cobb-Douglas technology that yields a unit measure of aggregate loan services. As a result, the asset composition of banks is given by loans in domestic and foreign currency in addition to holdings of bonds issued by the central bank for sterilization purposes. On the other hand, we assume that household are allowed to hold deposits with banks that are denominated in domestic and foreign currency. However, we introduce limits on household foreign currency denominated deposits by assuming transaction costs as a simple way to capture incomplete arbitrage.

Second, the severity of the bank's agency problem depends directly on a measure of currency mismatch at the bank level given by the difference between dollar denominated liabilities and assets as a fraction of total assets. However, not all assets enter symmetrically into the banks' incentive compatibility constraint that characterizes the agency problem. In particular, central bank assets are harder to divert than private loans. Third, the central bank "leans against the wind" regarding exchange rate pressures due to external shocks, but in a sterilized manner. In our setting, an FX intervention is a balance sheet operation that takes place when the central bank sells dollars to, or buys dollars from, the banking system in exchange for domestic currency-denominated assets. However, it does so in a way that completely offsets any change in the supply of domestic liquidity by using domestic bonds issued by the central bank.

Accordingly, the model predicts the existence of different interest rate spreads (excess returns) that limit banks' ability to borrow. When the incentive constraint binds and households face limited participation in foreign currency deposits, not only the return on banks' assets exceeds the return on deposits, including the excess return to foreign currency-denominated loans, but also the return on domestic currency-denominated deposits exceeds the return on foreign currency-denominated liabilities. Consequently, when financial frictions are active, the model predicts deviations from the standard uncovered interest rate parity equation: banks would be willing to borrow more from households and from international financial markets in foreign currency while households are unable to engage in frictionless arbitrage of foreign currency-denominated deposit returns.

In this setting, we study the transmission of external shocks on domestic financial conditions by assessing the role of FX interventions to “lean against the wind” with respect to exchange rate fluctuations and stabilize the response of interest rate spreads and bank lending. External shocks are transmitted to the domestic economy through changes in the exchange rate, interest rate spreads, and banks’ net worth. FX interventions are non-neutral when limits to arbitrage are present for banks and households.

For example, a persistent commodity boom generates a domestic economic expansion that, among other things, rises commodity exports significantly. A large fraction of the revenues from commodity exports is kept in the economy, causing a persistent exchange rate appreciation that less than partially offsets the impact on net exports due to a fall in non-commodity exports. The exchange rate appreciation relaxes the agency problem that banks face by increasing the net worth and the intermediation capacity of banks, which after the shock are less exposed to foreign currency liabilities. The latter effect is reinforced by a persistent decline in the banking system currency mismatch that feeds back to relax the financial constraint even more. By the same token, the interest rate spreads of banks’ assets over deposits move towards inducing banks to lend more in both currencies. It is noticeable that the persistent exchange rate appreciation increases credit dollarization but reduces deposit dollarization.

When FX interventions are active, the central bank builds FX reserves and allocates central bank riskless bonds to the banking system as a response to commodity booms. Given the binding agency problem, building FX reserves after a persistent increase in commodity prices significantly reduces exchange rate appreciation as well as the responses of currency mismatch and banks’ net worth. Thereby, limiting bank credit growth and the consequent expansion of macroeconomic aggregates such as consumption and investment. Besides exchange rate stabilization and its direct effects on intermediation, our framework implies an additional channel for FX interventions associated with the sterilization process. The associated sterilization operation increases the supply of central bank bonds to be absorbed by banks. The latter generates a crowding-out effect in banks’ balance sheets that reduces bank intermediation. Note that both effects are consistent with the empirical findings in Hofmann et al. (2019). Consequently, FX interventions present two potential transmission mechanisms in our framework: 1) the exchange rate smoothing channel and 2) the balance sheet substitution channel. The former channel affects the size of the currency mismatch at the bank level while the latter works through the availability of bank resources to extend loans.

We take the model to the data to quantify the transmission mechanism of external shocks and the role of FX interventions in mitigating their impact on the domestic economy. We consider commodity price shocks as described above, but also shocks on the foreign interest rate and global GDP. This exercise is intended to quantify the differences in the response of the economy to external shocks when FX interventions are activated, compared to exchange rate flexibility. We also conduct a standard welfare exercise to analyze whether FX interventions yield welfare gains in the presence of external shocks.

Recent empirical evidence show that our framework is general enough to be consistent with the experience of many EMEs facing frequent external shocks under a managed exchange rate regime along with banking systems characterized with significant financial dollarization and currency mismatch. On one hand, Levi-Yeyati and Sturzenegger (2016) classify the exchange rate regime of emerging market and advanced economies based on a ”de facto” criteria, and find that, more than half of the countries in their sample, adopt a non-floating exchange rate regime. Based on the same criteria, Aguirre et al 2019 report that none of the countries that have implemented IT since 1991 have always kept a purely floating exchange rate regime. Moreover, periods during which several countries (reaching around 60% of them) where non-pure floaters coincide with events related to external fundamentals. On the other hand, Corrales and Imam (2019) examine countries from different regions using the International Financial Statistics database from 2001 to 2016 and report that households maintain 57.5 percent of their deposits in dollars, while for firms, 68.7 of their loans are denominated in dollars. Castillo et al (2019) study 45 emerging market and advanced economies, excluding countries whose central bank issue a reserve currency and report that around 50 percent of the countries in their sample are classified as dollarized economies. Moreover, the authors show that dollarized economies experience larger macroeconomic volatility in response to global capital flows relative to non-dollarized economies and find that active FX interventions successfully reduce output and exchange rate volatility to global capital inflows.

Our quantitative analysis uses data for the Peruvian economy since it is representative of EMEs under an inflation targeting regime with FX interventions, financial dollarization, and a commodity exporter small open economy facing external shocks continuously. We consider that using data for several EMEs instead, maybe misleading since evidence also shows that there is a high degree of heterogeneity in the strategies, instruments, and tactics used to implement FX intervention policies (see Hendrick et all 2019). Therefore, we calibrate most of the parameters associated with the banking block of the model to replicate some financial steady-state targets for Peru's banking system. The rest of the parameterization is done by matching the impulse responses of the economic model to the impulse responses implied by an SVAR model with block exogeneity under the small open economy assumption.

Quantitatively, our results suggest that FX interventions successfully reduce macroeconomic volatility to external shocks (notably credit, investment, and output unconditional volatilities decrease by around 82%, 65%, and 70%, respectively when compared to exchange rate flexibility). Moreover, conditional on an increase of 20 basis points in the foreign interest rate, a sterilized purchase of FX reserves reduces the relative two-year accumulated response of aggregate bank lending, investment and GDP by around 89%, 51% and 51%, respectively. Likewise, when the economy faces a commodity boom (an increase of 6.31% in the commodity export index), a sterilized purchase of FX reserves limits the two-year commodity price relative accumulated response of bank lending from around 0.33 to 0.02. Consequently, the response of investment and GDP is also muted by a 63% and 60% respectively. Hence, our quantitative results are indicative that FX intervention might create significant welfare gains in responding to external shocks. Using a standard welfare analysis, we find that if the central bank does not intervene in the FX market in the face of external shocks, there would be a welfare loss of 6.2% in consumption, given the standard parameterization of the Taylor rule for the conventional interest rate instrument.

Furthermore, we explore additional numerical experiments. We recalibrate the steady state of the model economy to be consistent with a higher steady state level for the average currency mismatch of the banking system. We consider an increase of five additional percentage points relative to our baseline calibration by targeting a lower foreign interest rate and a higher level of central bank bonds at the steady state. These new targets induce banks to be more exposed to potential currency mismatches. Not surprisingly, our results suggest that FX interventions are more effective when the economy is calibrated to be consistent with a higher level of currency mismatch at the steady state since banks are in a more vulnerable initial position with respect to external shocks that produce unexpected depreciations.

Then we relax three assumptions of our basic formulation of the model that may be viewed as strong and restrictive with the aim to study our setting under more general assumptions.

First, we consider the case of an economy without financial dollarization where intermediate good producers borrow from banks only in domestic currency and households are not allowed to hold deposits with banks that are denominated in foreign currency. Consequently, banks lend only in domestic currency while the only source of foreign currency funding for banks comes from borrowing abroad. In the steady state equilibrium banks are more exposed to real exchange rate movements while non-financial firms as well as households are less exposed to these fluctuations. Our parametrization suggests that when the economy is not financially dollarized, FX interventions are still not neutral but less effective than in the financially dollarized economy in smoothing the response of the exchange rate as well as the response of financial and macroeconomic variables to external shocks.

Second, we relax the limited participation assumption of households with respect to bank deposits denominated in foreign currency by assuming a limited case of zero transaction costs. Consequently, household's demand for bank deposits in foreign currency is infinitely responsive to arbitrage opportunities implying that in equilibrium the UIP condition holds with a constant premium while the incentive compatibility constraint for banks is still binding. Our simulations show that in this case, the exchange rate smoothing channel of FX interventions is not active, nevertheless the sterilization process associated to FX interventions presents a relatively small effect over financial and macroeconomic variables due to the balance sheet substitution channel. In our model, for FX interventions to affect significantly the real exchange rate and excess returns along with the aggregate equilibrium of the economy, limits to arbitrage between domestic and foreign currency denominated assets and liabilities must be present for both, households and banks.

Finally, in the last extension of the model, the severity of the bank's agency problem depends directly on an industry (aggregate) measure of currency mismatch instead than on an individual measure. In this case, banks do not internalize the effects of borrowing and lending in foreign currency on the aggregate currency mismatch of the banking system. As a result, banks are indifferent between borrowing from domestic depositors and from abroad, implying that the standard UIP condition holds without any endogenous risk premium. Notably in this case, even though the incentive constraint for banks binds the response of the real exchange rate to external shocks is the same under FX interventions and exchange rate flexibility. This result differs from Céspedes et al. (2017) and Chang (2019) where FX interventions are irrelevant only when the incentive compatibility constraint does not bind. In this extension, the associated sterilization operation generates negligible real effects for several macroeconomic variables relative to our baseline case. Thus, in terms of macroeconomic variables different from the real exchange rate, FX interventions are almost neutral in this case. Our irrelevance result is due to the indeterminacy of banks' liability composition that occurs when banks do not internalize the effect of currency mismatch over financial constraints. Furthermore, we simulate an exogenous purchase of FX reserves under the last two extensions of the model and find that FX interventions are irrelevant for real exchange rate dynamics even when the incentive compatibility constraint binds.

The remainder of the paper is organized as follows. Section 1 briefly reviews the literature related to FX interventions in macroeconomic models. Section 2 describes the general equilibrium model with a special emphasis in the financial system and the implementation of FX interventions. Section 3 presents the parametrization strategy, including the specification and identification assumptions for the SVAR model. The main results are shown in Section 4. Section 4.4 studies the effects of external shocks on some generalizations of our basic formulation of the model. Finally, Section 6 concludes with some final remarks.

1 Brief Literature Review

We divide the literature about FX interventions into three broad stages. Pioneered by Kouri (1976), Branson et al. (1977), and Henderson and Rogoff (1982), the first strand of this literature emphasizes the portfolio balance channel, which indicates that, when domestic and foreign assets are imperfect substitutes, FX intervention is an additional and effective central bank tool. This is because it can change the relative stock of assets and with it the exchange rate risk premium that affects arbitrage possibilities between the rates of return of domestic currency denominated assets and foreign currency denominated assets. However, the models built during this stage were characterized by a lack of solid micro-foundations, preventing a rigorous normative analysis. Additional research studies within the portfolio balance approach without micro-foundations are Krugman (1981), Obstfeld (1983), Dornbusch (1980), Branson and Henderson (1985), and Frenkel and Mussa (1985).

Relying on micro-founded general equilibrium models, the second strand of this literature states that FX interventions have no effect on equilibrium prices and quantities. The seminal work using this approach is Backus and Kehoe (1989), which not only studies the effectiveness of this kind of intervention in complete markets, but also considering some types of market incompleteness. It points out that, when portfolio decisions are frictionless, the imperfect substitutability between domestic and foreign assets postulated by the portfolio balance channel is not enough for FX interventions to affect prices and quantities in the general equilibrium. After the publication of this work, academia adopted a pessimistic view with respect to the effectiveness of FX interventions, creating a long-lasting dissonance with policy practice since policy-makers have ignored the recommendations from research and have intervened, frequently and intensely, in the FX market.

Recently, there has been a resurgence in academic interest in assessing the relevance of FX interventions based on micro-founded macroeconomic models. In this regard, the portfolio balance approach has experienced a recent comeback in studies such as Kumhof (2010), Gabaix and Maggiori (2015), Liu and Spiegel (2015), Benes et al. (2015), Montoro and Ortiz (2016), Cavallino (2019), and Castillo et al. (2019). Some of these studies rely on a reduced form type of friction while others assume more structure when addressing the relevance of FX interventions. This literature argues that FX intervention can affect the exchange rate when domestic and external assets are imperfect substitutes. In this case, FX intervention increases the relative supply of domestic assets, driving the risk premium up and creating exchange rate depreciation pressures.

A third strand of the literature is the so-called financial intermediation view of FX interventions. The general equilibrium relevance of FX interventions rely on a financial friction of the type associated with the literature on unconventional monetary policy in closed economies. Specifically, this literature assumes that banks face an agency problem that constraints their ability to obtain funds from abroad. Céspedes et al. (2017) and Chang (2019) build models for an open economy with domestic banks subject to occasionally binding collateral constraints and find that FX interventions have an impact on macroeconomic aggregates only when the relevant financial constraint is binding. When financial markets are frictionless, domestic banks are able to accommodate FX interventions by borrowing less or more from domestic depositors as well as from foreign financial markets. In the latter case, the general equilibrium is left undisrupted. Additionally, Fanelli and Straub (2019) find that including a pecuniary externality in partially segmented domestic and foreign bond markets results in an excessively volatile exchange rate response to capital inflows, thereby making FX interventions desirable.

Empirical evidence on the effectiveness of FX interventions has been particularly difficult to find because of endogeneity problems that make it difficult to identify its effects, especially on the exchange rate. While individual country studies report mixed results on the effectiveness of FX intervention, in general cross-country studies find some effectiveness in curbing financial conditions and exchange rate dynamics (see Ghosh et al. (2018), Villamizar-Villegas and Perez-Reyna (2017), and Fratzscher et al. (2018). Recent empirical findings have shed some light on how FX intervention reduces the impact of capital flows on domestic financial conditions. For instance, Blanchard et al. (2015) show that capital flow shocks have significantly smaller effects on exchange rates and capital accounts in countries that intervene in FX markets on a regular basis. According to Hofmann et al. (2019), FX intervention has two mutually reinforcing effects. On one hand, in periods of easing global financial conditions, FX can be used to lean against the increase in bank lending after a dollar appreciation (the risk-taking channel of the exchange rate). On the other hand, there is a "crowding out" effect of bank lending associated to the sterilization process of the FX intervention, which increases the supply of domestic bonds absorbed by banks. The aggregate impact of FX interventions results from the mix of these two effects. By curbing domestic credit, FX intervention will have an impact on the real economy.

2 A General Equilibrium Model

We build a medium-scale small open economy New Keynesian model extended with banks, FX interventions, and a commodity sector. Following ABK, banks are allowed to finance their assets using two kinds of liabilities: domestic deposits and foreign borrowing from international financial markets. Nevertheless, banks lend not only in domestic currency but also in FX. FX intervention is introduced to study the role of this tool in financial intermediation, macroeconomic stabilization, and exchange rate volatility.

The rest of the model follows very closely the standard small open economy New Keynesian framework with the exception of two main features. First, we introduce an endogenous commodity sector to analyze the effect of commodity booms and busts in domestic financial conditions. The representative commodity producer accumulates its own capital facing standard capital adjustment costs and does not need external funding or any form of borrowing to produce. Second, we assume that intermediate good producers must borrow from banks before producing. In addition, we assume that intermediate good producers demand a bundle of loans consisting of a combination of domestic and foreign currency denominated loans according to a loan services technology that aggregates both types of loans. Further details about the model are presented below. For the rest of the document, small letters characterize individual variables, while capital letters denote aggregates.

2.1 The Financial System

We follow Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) to introduce a banking sector in an otherwise standard infinite horizon macroeconomic model for a small open economy. In this setting, the representative household consists of a continuum of bankers and workers of measure unity. Workers supply labor and provide labor income to their households. Workers hold deposits with banks along with private securities in the form of equity with intermediate good producers. Domestic bank deposits are denominated in domestic and foreign currency, although the latter is subject to transaction costs. Foreign agents lend to banks in foreign currency and are precluded from lending directly to non-financial firms. All financial contracts between agents are short-term, non-contingent, and thus riskless. An agency problem constraints banks' ability to obtain funds from households and foreigners. The tightness of the financial constraint that banks face depends on a measure of currency mismatch at the individual level. In this section, we focus on bankers, while workers are described in detail in section 2.3.

Banks. In a given household, each banker member manages a bank until she retires with probability $1-\sigma$. Retired bankers transfer their earnings back to households in the form of dividends and are replaced by an equal number of workers that randomly become bankers. The relative proportion of bankers and workers is kept constant. New bankers receive a fraction ξ of total assets from the household as start-up funds.

Additionally, banks provide funding to producing firms without any financial friction. Hence, the only financially constrained agents in the model are banks due to a moral hazard problem between a bank and its depositors.[4] Domestic and foreign currency denominated bank loans to firms are denoted by l_t and l_t^*, respectively. Bank assets are also made up of central bank bonds (b_t) considered to be the only financial instruments used in the associated sterilization process of any FX intervention. Bank investments are financed by domestic currency-denominated household deposits (d_t), by foreign currency-denominated household deposits ($d_t^{*,h}$), by foreign borrowing ($d_t^{*,f}$), or by using banks' own net worth (n_t). A bank's balance sheet expressed in real terms is

$$l_t + e_t l_t^* + b_t = n_t + d_t + e_t(\overbrace{d_t^{*,h} + d_t^{*,f}}^{d_t^*}) \tag{1}$$

where e_t is the real exchange rate. Table 1 illustrates the typical balance sheet of a bank in the model.

Table 1. Bank's Balance Sheet

Assets	**Liabilities**
l_t	d_t
$e_t l_t^*$	$e_t(d_t^{*,h} + d_t^{*,f})$
b_t	n_t

We assume that $d_t^{*,h}$ and $d_t^{*,f}$ are perfect substitutes for bankers and d_t^* denotes total deposits/funding in foreign currency. Net worth is accumulated through retained earnings and it is defined as the difference between the gross return on assets and the cost of liabilities:

$$n_{t+1} = R_{t+1}^l l_t + R_{t+1}^{l*} e_{t+1} l_t^* + R_{t+1}^b b_t - R_{t+1} d_t - e_{t+1} R_{t+1}^* d_t^* \tag{2}$$

where $\{R_t^b,\ R_t^l,\ R_t^{l*}\}$ denote the real gross returns to the bank from central bank bonds, domestic currency-denominated loans, and foreign currency-denominated loans, respectively. Similarly, R_t and R_t^* are the real gross interest rate paid by the bank on domestic and foreign currency- denominated liabilities, respectively.[5]

Agency Problem. With the purpose of limiting banks' ability to raise domestic and foreign funds, we assume that at the beginning of the period, bankers may choose to divert funds

[4] Households face limited participation in asset markets when saving in foreign currency and holding equity. Limited participation appears in terms of a marginal transaction cost for managing sophisticated portfolios.

[5] All real interest rates are ex-post. Along these lines, R_t equals $\frac{1+i_{t-1}}{1+\pi_t}$ where i_t is the nominal policy rate.

from the assets they hold and transfer the proceeds to their own households. If bank managers operate honestly, then assets will be held until payoffs are realized in the next period and repay their liabilities to creditors (domestic and foreign). On the contrary, if bank managers decide to divert funds, then assets will be secretly channeled away from investment and consumed by their households. In this framework, it is optimal for bank managers to retain earnings until exiting the industry. Bankers' objective is to maximize the expected discounted stream of profits that are transferred back to the household; i.e., its expected terminal wealth, given by

$$V_t = \mathbb{E}_t \left[\sum_{j=1}^{\infty} \Lambda_{t,t+j} \sigma^{j-1} (1-\sigma) n_{t+j} \right]$$

where $\Lambda_{t,t+j}$ is the stochastic discount factor of the representative household from $t+j$ to t and $\mathbb{E}_t[.]$ is the expectation operator conditional on information set at t. Notice that using $\Lambda_{t,t+j}$ to properly discount the stream of bank profits means that households effectively own the banks that their banker members manage. Bank managers will abscond funds if the amount they are capable to divert exceeds the continuation value of the bank V_t. Accordingly, for creditors to be willing to supply funds to the banker, any financial arrangement between them must satisfy the following incentive constraint:

$$V_t \geq \Theta(x_t) \left[l_t + \varpi^* e_t l_t^* + \varpi^b b_t \right] \tag{3}$$

where $\Theta_t(x)$ is assumed to be strictly increasing[6] and x_t is the currency mismatch measure at he bank level defined and discussed below. We assume that some assets are more difficult to divert than others. Specifically, a banker can divert a fraction $\Theta(x_t)$ of domestic currency loans, a fraction $\Theta(x_t)\varpi^*$ of foreign currency loans, and a fraction $\Theta(x_t)\varpi^b$ of the total amount of central banks bonds, where $\varpi^*, \varpi^b \in [0, \infty)$. For instance, whenever $\varpi^b = 0$, bankers cannot divert sterilized bonds and buying them does not tighten the incentive constraint. Therefore, a fraction of the interest rate spread on b_t may be arbitraged away, leaving R_t^b lower than R_t^l. In our setting, the three type of assets held by banks do not enter with equal weights into the incentive constraint, reflecting that for some assets the constraint on arbitrage is weaker. We calibrate ϖ^*, and ϖ^b to match the average gross returns for each asset type in the Peruvian economy. In Section 3, we show that those targets are consistent with the fact that central bank bonds are much harder to divert than loans; i.e., the calibrated ϖ^b is very close to zero. In Section 4.4 we relax this assumption and assume that all assets enter the incentive constraint with equal weights.

We assume that the banker's ability to divert funds depends on the currency mismatch size at the bank level expressed as a fraction of total assets. In this regard, we define x_t to be

$$x_t = \frac{e_t d_t^* - e_t l_t^*}{l_t + e_t l_t^* + b_t} \tag{4}$$

A higher currency mismatch size at the bank level implies that bankers are able to divert a higher fraction of their assets, ultimately increasing the severity of the incentive constraint. In this regard, x_t measures the exposure of the bank's balance sheet to abrupt exchange rate movements and foreign capital reversals. A significant currency mismatch degree in a bank's balance sheet places it in a more vulnerable position with respect to external shocks,

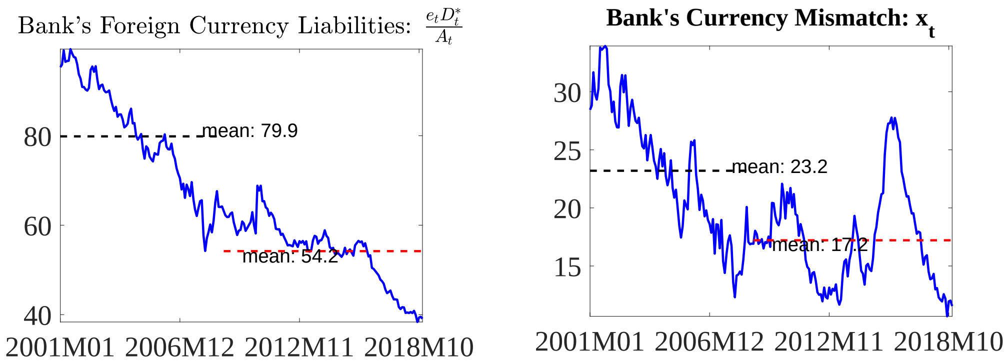
[6] Specifically, we use the following convex function:

$$\Theta(x) = \theta \left(1 + \frac{\varkappa}{2} x^2 \right)$$

particularly shocks generating unexpected depreciations. From this perspective, and as long as the incentive constraint is binding, an increase in x_t will require an increase in V_t, to keep domestic depositors and foreign lenders willing to continue lending funds to a bank. In the basic formulation of the model, we assume that x_t is internalized by each bank. In Section 4.4, we assume that x_t is external to an individual bank representing an aggregate currency mismatch measure of the banking system as a whole.

Figure 1 plots both the evolution of foreign currency liabilities and the currency mismatch level of Peru's banking system.[7] Foreign currency deposits, including external credit lines, as a fraction of total assets have been steadily decreasing since 2001, from an average of 79.9% during 2001-2008 to an average of 54.2% ever since.This is also the case for the empirical measure of currency mismatch showing a decreasing trend and an average of 23 percent during 2001-2008. From 2009 to 2018, it has been fluctuating around 17.2% without showing a clear trend. In Section 3, we use this data set to discipline the model.

FIGURE 1. CURRENCY MISMATCH IN PERUVIAN DATA, %



Bank's Recursive problem. Given a function $\Theta(x)$, a vector of interest rates, government policies, and n_t (state variable), each bank chooses its balance sheet components $(l_t, l_t^*, b_t, d_t, d_t^*)$ to maximize the franchise value:

$$V_t = \max_{l_t, l_t^*, b_t, d_t, d_t^*} \mathbb{E}_t \left[\Lambda_{t,t+1} \left\{ (1-\sigma) n_{t+1} + \sigma V_{t+1} \right\} \right]$$

subject to (1), (2), (3), and (4).

A bank's objective function as well as its balance sheet and the incentive constraint it faces, can be expressed as a fraction of net worth. Moreover, using the definition of x_t, a bank's problem can be written in terms of choosing each of the assets it holds as a fraction of net worth together with the optimal size of its currency mismatch x_t. Consequently, the bank's problem is to choose (ϕ_t, ϕ_t^*, ϕ_t^b, x_t) to maximize its value as a fraction of net worth:

$$\psi_t = \max_{\phi_t^l, \phi_t^{l*} \phi_t^b, x_t} \quad \mu_t^l \phi_t^l + (\mu_t^{l*} + \mu_t^{d*}) \phi_t^{l*} + \mu_t^b \phi_t^b + \mu_t^{d*} \left(\phi_t^l + \phi_t^{l*} + \phi_t^b \right) x_t + \upsilon_t \tag{5}$$

subject to:

$$\psi_t - \Theta(x_t) \left[\phi_t^l + \varpi^* \phi_t^{l*} + \varpi^b \phi_t^b \right] \geq 0 \tag{6}$$

[7] We calibrate the consolidated balance sheet of the banking system in the model using data for Peru to obtain historical averages for the aggregate currency mismatch level and foreign currency liabilities as a fraction of total assets. We use data on domestic currency credit for L_t, foreign currency - denominated liabilities for L_t^* and total banking investments for B_t. Additionally, we use data on banks' net worth for N_t and the sum of foreign currency deposits and external liabilities for measuring D_t^*.

where $\psi_t = \frac{V_t}{n_t}$, $\phi_t = \frac{l_t}{n_t}$, $\phi_t^* = \frac{e_t l_t^*}{n_t}$, $\phi_t^b = \frac{b_t}{n_t}$, $v_t = \mathbb{E}_t\left[\Omega_{t+1} R_{t+1}\right]$, and

$$\mu_t^l = \mathbb{E}_t\left[\Omega_{t+1}\left(R_{t+1}^l - R_{t+1}\right)\right]$$
$$\mu_t^{l*} = \mathbb{E}_t\left[\Omega_{t+1}\left(\frac{e_{t+1}}{e_t} R_{t+1}^{l*} - R_{t+1}\right)\right]$$
$$\mu_t^b = \mathbb{E}_t\left[\Omega_{t+1}\left(R_{t+1}^b - R_{t+1}\right)\right]$$
$$\mu_t^{d*} = \mathbb{E}_t\left[\Omega_{t+1}\left(R_{t+1} - \frac{e_{t+1}}{e_t} R_{t+1}^*\right)\right]$$

Ω_{t+1} is the shadow value of a unit of net worth to the bank at $t+1$, given by

$$\Omega_{t+1} = \Lambda_{t,t+1}(1 - \sigma + \sigma\psi_{t+1})$$

Let λ_t^b be the Lagrangian multiplier for the incentive constraint faced by the bank, eq. (6). Then, the first order conditions are characterized by the slackness condition associated to eq. (6) and:[8]

$$\mu_t^l + \mu_t^{d*} x_t = \frac{\lambda_t^b}{1+\lambda_t^b}\Theta(x_t) \tag{7}$$

$$\mu_t^{l*} + \mu_t^{d*}(1 + x_t) = \frac{\lambda_t^b}{1+\lambda_t^b}\varpi^*\Theta(x_t) \tag{8}$$

$$\mu_t^b + \mu_t^{d*} x_t = \frac{\lambda_t^b}{1+\lambda_t^b}\varpi^b\Theta(x_t) \tag{9}$$

$$\mu_t^{d*}\left(\phi_t^l + \phi_t^{l*} + \phi_t^b\right) = \frac{\lambda_t^b}{1+\lambda_t^b}\left(\phi_t^l + \varpi^*\phi_t^{l*} + \varpi^b\phi_t^b\right)\frac{\partial\Theta(x_t)}{\partial x} \tag{10}$$

When the incentive constraint is not binding, then $\lambda_t^b = 0$, the discounted excess returns or interest rate spreads are zero. Consequently, under this equilibrium, financial markets are frictionless implying that the standard arbitrage condition holds: banks will acquire assets to the point where the discounted return on each asset equals the discounted cost of deposits (i.e., $\mu_t^l = \mu_t^{l*} = \mu_t^b = 0$). In addition, there is no cost advantage of foreign borrowing over domestic deposits (i.e., $\mu_t^{d*} = 0$, the UIP conditions holds).

When the incentive constraint is binding, $\lambda_t^b > 0$, banks are restricted to obtain funds from creditors. In this context, limits to arbitrage emerge in equilibrium, leading to interest rate spreads. It is important to highlight that excess returns increase depending on how tightly the incentive constraint binds. The latter is measured by λ_t^b and ultimately depends on x_t. The intuition behind the above first-order conditions is that banks invest in each asset to the point where the marginal benefit of acquiring an additional unit of each asset is equal to its marginal cost. The marginal benefit of each asset is composed by its own discounted excess value and the excess value associated with the advantage cost of funding it via foreign borrowing, which is ultimately influenced by the size of the currency mismatch[9]. For instance,

[8] A complete derivation of the bank's optimality conditions are presented in Appendix C.1.

[9] Note that the marginal benefit for each asset can be rewritten in terms of interest rate spreads as

$$\mu_t^l + \mu_t^{d*} x_t = \mathbb{E}_t\left[\Omega_{t+1}\left(R_{t+1}^l - \left\{\frac{e_{t+1}}{e_t} R_{t+1}^* x_t + R_{t+1}(1 - x_t)\right\}\right)\right]$$
$$\mu_t^b + \mu_t^{d*} x_t = \mathbb{E}_t\left[\Omega_{t+1}\left(R_{t+1}^b - \left\{\frac{e_{t+1}}{e_t} R_{t+1}^* x_t + R_{t+1}(1 - x_t)\right\}\right)\right]$$
$$\mu_t^{l*} + \mu_t^{d*}(1 + x_t) = \mathbb{E}_t\left[\Omega_{t+1}\left(R_{t+1}^{l*} - \left\{\frac{e_{t+1}}{e_t} R_{t+1}^*(1 + x_t) + R_{t+1}(-x_t)\right\}\right)\right]$$

Then, it is clear that x_t directly influences the fraction of each asset financed by foreign currency borrowing.

a fraction x_t of an extra unit of l_t or b_t is funded by d_t^*. Similarly, a portion $1 + x_t$ of an additional investment in l_t^* is financed by d_t^*; i.e., banks use more foreign currency funds and less home deposits per unit of foreign currency loans. On the other hand, the marginal cost associated with each asset is given by the marginal cost of tightening the incentive constraint times the total share of the asset that the bank may actually divert.

Limits to arbitrage emerge from the restriction that the incentive constraint places on the size of a bank's portfolio relative to its net worth. A form of leverage ratio for a bank can be obtained by combining eq. (5), eq. (6), and the above first order conditions,

$$\Phi_t n_t \geq l_t + \varpi^* e_t l_t^* + \varpi^b b_t \tag{11}$$

$$\Phi_t = \frac{\upsilon_t}{\Theta(x_t) - \left(\mu_t^l + \mu_t^{d*} x_t\right)} \tag{12}$$

Gertler and Karadi (2013) argued that Φ_t can be interpreted as the maximum ratio of weighted assets to net worth that a bank may hold without violating the incentive constraint. The weight applied to each asset is the proportion of the asset that the bank is able to divert.

When the incentive constraint binds, the weighted leverage ratio Φ_t is increasing in two factors: 1) the savings of deposit costs from another unit of net worth given by υ_t; and 2) the discounted marginal benefit of lending in domestic currency. As discussed in Gertler et al. (2012), both factors raise the value of a bank, thereby making its creditors willing to lend more. The leverage ratio also varies inversely with exchange risk perceptions ultimately associated to fluctuations on x_t: whenever the currency mismatch rises, bankers are more exposed to real exchange movements and its creditors restrict external funding. Notice that in a closed economy setting, μ_t^{d*} is zero and Φ_t constant. In this case, eq. (12) converges to the setup for a bank's leverage ratio proposed by Gertler and Karadi (2013).

The leverage ratio can be expressed as a collateral constraint consistent with Kiyotaki and Moore (1997) as follows:

$$l_t \leq \theta_t n_t \qquad \text{and} \qquad \theta_t = \Phi_t - \varpi^* \phi_t^* - \varpi^b \phi_t^b$$

where $\phi_t^* = \frac{e_t l_t}{n_t}$ and $\phi_t^b = \frac{b_t}{n_t}$. Recently, Céspedes et al. (2017) and Chang (2019) use similar collateral constraints to capture foreign debt limits faced by EME domestic banks. However, in our more general framework, θ_t is not a parameter but an endogenous variable that depends on a currency mismatch measure at the bank level. In our setting, similar collateral constraints for l_t^* and b_t can be obtained straightforwardly[10].

2.2 The Central Bank and FX Interventions

The related literature on FX intervention (for example, Chang (2019)) agrees in defining it as the following situation: whenever a central bank sells or buys FX and at the same time it also buys or sells an equivalent amount of domestic currency-denominated securities. Under this policy, the central bank's net credit position changes. Without sterilization, buying or selling FX would directly affect the supply of domestic liquidity. The latter implies difficulties in

[10] These collateral constraints are:

$$e_t l_t^* \leq \theta_t^* n_t \qquad \text{and} \qquad \theta_t^* = \frac{\Phi_t}{\varpi^*} - \frac{1}{\varpi^*}\phi_t^l - \frac{\varpi^b}{\varpi^*}\phi_t^b$$

$$b_t \leq \theta_t^b n_t \qquad \text{and} \qquad \theta_t^b = \frac{\Phi_t}{\varpi^b} - \frac{\varpi^*}{\varpi^b}\phi_t^* - \frac{1}{\varpi^b}\phi_t^l$$

meeting the central bank's interbank interest rate target, which ultimately is determined by a Taylor rule. Nevertheless, there is less agreement in the literature about the implementation of the sterilization leg of an FX intervention. This reflects differences in FX intervention practices among central banks.

In our framework, the sterilization operations associated with an FX intervention are implemented by changing the supply of central bank bonds in the banking system. Recall that central bank bonds are riskless one-period bonds issued by the monetary authority. Accordingly, FX intervention denotes the following: if the central bank buys (sells) FX, for example dollars, from (to) the domestic banking system, a simultaneous raise (fall) in official FX reserves would occur. At the same time, the central bank will completely offset the effect on domestic liquidity by issuing (retiring) central bank bonds to (from) the banking system. The central bank's balance sheet is given by

$$B_t = e_t F_t \tag{13}$$

where B_t denotes central bank bonds and F_t official FX reserves. Notice that eq. (13) serves both as a sterilization rule and as accounting identity for the central bank's balance sheet. In this setting, FX interventions induce the central bank to produce operational losses or a quasi-fiscal deficit, since it is assumed that official FX reserves are invested abroad at the foreign interest rate R_t^*, while central bank bonds pay R_t^b. Then, the central bank's quasi-fiscal deficit is:

$$CB_t = \left(\tau^{fx} + R_t^b - \frac{e_t}{e_{t-1}} R_t^*\right) B_{t-1} \tag{14}$$

where τ^{fx} measures a inefficiency cost for FX intervention which plays a main role in the welfare analysis of the model (see Section 5). As long as $R_t^b > R_t^*$, the central bank produce operational losses associated with the sterilization process, which ultimately represent the fiscal costs of FX interventions. We assume that any operational losses are transferred to the central government and financed through lump sum taxes on households.

Furthermore, in addition to the standard policy rate rule, the central bank implements the following FX intervention rule written in terms of the supply of central bank bonds responding to exchange rate deviations from its steady-state value:

$$\ln B_t = (1 - \rho_B) \ln B + \rho_B \ln B_{t-1} - \upsilon_e (\ln e_t - \ln e) \tag{15}$$

with $\upsilon_e > 0$ and $0 < \rho_B < 1$ measure the intensity with which FX interventions respond to exchange rate movements and its persistence, respectively. The steady-state level of central bank bonds is denoted by B. Under this rule, the central bank sells official FX reserves in response to a real depreciation (i.e., whenever the real exchange rate is above its steady state value). As mentioned before, the counterpart of selling reserves is to withdraw central bank bonds from banks' balance sheet, eq. (13). Consequently, FX interventions present two potential transmission mechanisms in our framework: 1) when selling official FX reserves to the banking system, the exchange rate is stabilized; and 2) when sterilizing the effect over domestic liquidity, the central bank frees resources from domestic banks to extend additional loans to firms. Moreover, the exchange rate stabilization effect potentially affects the size of the currency mismatch size at the bank level. For instance, ceteris paribus, stabilizing a depreciation pressure on the exchange rate may lead to reducing the currency mismatch size at the bank level. If this is the case, the incentive constraint (more specifically, its degree of tightening) may be relaxed even further, thereby further stimulating domestic financial conditions.

One key aspect of our model is that FX interventions are relevant for determining the general equilibrium allocation only when the incentive constraint binds, as in Céspedes et al. (2017) and Chang (2019). Whenever the incentive constraint is not binding, financial markets are frictionless, meaning there is no leverage constraint for banks nor interest rate spreads. Therefore, balance sheet policies such as FX interventions are irrelevant, since the size and composition of balance sheets, for both the banking system and the central bank, do not matter for equilibrium. In particular, under frictionless financial markets, the sterilization process associated with FX interventions does not have real effects: the exchange rate, as well as domestic financial conditions, are determined without any consideration of balance sheets. More important, in our framework, and in contrast with Chang (2019), domestic banks can accommodate the central bank's FX reserve accumulation during "normal" times (non-binding incentive constraint) by increasing domestic deposits, foreign borrowing, or both, since banks are indifferent between domestic-currency or foreign currency funding. Therefore, when the incentive constraint is not binding and the central bank accumulates FX reserves it does not necessarily mean that banks will end up more exposed to foreign currency-denominated liabilities. Furthermore, in Section 4.4, we consider an extension of our baseline model where banks take as given fluctuations in x_t. In this case, banks consider domestic deposits and foreign borrowing as perfect substitutes, the UIP condition holds with equality and FX interventions are irrelevant for exchange rate dynamics even though the incentive constraint binds.

We consider that for EME's, financial constraints are always binding, even in "normal" times. The difference between normal times and a financial crisis is how tight financial constraints bite. In our framework, the degree of financial constraint tightening depends on the currency mismatch size in banks' balance sheets, which ultimately responds to external shocks. In this context, FX interventions are meant to be an additional central bank instrument aimed to smooth the response of domestic financial conditions to external shocks via exchange rate stabilization.

2.3 Households

Workers supply labor and take labor income to their household. Households use labor income and profits from firm ownership to consume non-commodity goods, save by holding private securities issued by intermediate good producers along with bank deposits. As already mentioned, bank deposits by households are denominated in domestic and foreign currency. We assume that households face increasing transactions costs when holding equity along with foreign currency-denominated bank deposits. The latter assumption prevents frictionless arbitrage due to limited ability to manage sophisticated portfolios. Finally, in line with standard literature on financial and labor market frictions, it is assumed that within each household there is perfect consumption insurance to keep the representative agent assumption. Following Miao and Wang (2010) and Gertler et al. (2012), households' preference structure is

$$(1-\beta)\mathbb{E}_t\left[\sum_{j=0}^{\infty}\beta^j\frac{1}{1-\gamma}\left(C_{t+j}-\mathcal{H}C_{t+j-1}-\frac{\zeta_0}{1+\zeta}H_{t+j}^{1+\zeta}\right)^{1-\gamma}\right] \tag{16}$$

where C_t is consumption and H_t is the labor effort in terms of hours worked. The subjective discount factor is given by $\beta\in(0,1)$, $\gamma>0$, which measures the elasticity of intertemporal substitution, while ζ_0 controls the dis-utility of labor. Additionally, the Frisch elasticity is mainly determined by the interaction of $\zeta>0$ and the degree of internal habit formation, $\mathcal{H}\in[0,1)$. For instance, if there is no habit formation (i.e. $\mathcal{H}=0$), this specification abstracts from wealth effects on labor supply as in Greenwood et al. (1988), and the Frisch elasticity is $1/\zeta$.[11]

Bank deposits are assumed to be one-period riskless real assets that pay a gross real return of R_t from period $t-1$ to t. Let D_t and $D_t^{*,h}$ be the total quantity of domestic and foreign currency-denominated deposits, respectively. The amount of new equity acquired by the household is $\mathcal{S}_t$ while w_t denotes the real wage, R_t^{knc} the return on equity, Π_t is net payouts to the household from the ownership of both financial and non-financial firms and T_t denotes the lump-sum taxes needed to finance the central bank's quasifiscal deficit. Hence, the household budget constraint is written as

$$\begin{aligned} C_t + D_t + e_t\left[D_t^{*,h} + \frac{\kappa_{D*}}{2}\left(D_t^{*,h} - \overline{D}^{*,h}\right)^2\right] + \left[\mathcal{S}_t + \frac{\kappa_S}{2}\left(\mathcal{S}_t - \overline{\mathcal{S}}\right)^2\right] + T_t \\ = w_t H_t + \Pi_t + R_t D_{t-1} + R_t^* e_t D_{t-1}^{*,h} + R_t^{knc}\mathcal{S}_{t-1} \end{aligned} \tag{17}$$

where $(\kappa_{D*}, \overline{D}^{*,h})$ and $(\kappa_S, \overline{\mathcal{S}})$ are parameters that control the transaction costs for $D_t^{*,h}$ and $\mathcal{S}_t$, respectively. Accordingly, $\overline{D}^{*,h}$ and $\mathcal{S}$ correspond to the the frictionless capacity level for each asset. Consider the case where the marginal transaction cost is infinity. Then, households will hold the respective frictionless value of each asset, which is fully unresponsive to arbitrage opportunities. Notice that Π_t includes the net transfer to household members that become bankers at the beginning of the period, as it is written as

$$\begin{aligned} \Pi_t = & \underbrace{\Pi_t^1}_{\text{Goods Producer}} + \underbrace{\Pi_t^2}_{\text{Capital Producer}} + \underbrace{\Pi_t^c}_{\text{Commodity Sector}} \\ & + \underbrace{(1-\sigma)[R_t^l L_{t-1} + R_t^{l*} e_t L_{t-1}^* + R_t^b B_{t-1} - R_t D_{t-1} - R_t^* e_t D_{t-1}^*]}_{\text{Retiring bankers}} \\ & - \underbrace{\xi\left(R_t^l L_{t-1} + R_t^{l*} e_t L_{t-1}^* + R_t^b B_{t-1}\right)}_{\text{Bankers' start-up funds}} \end{aligned}$$

Hence, the representative worker chooses consumption, labor supply, and bank deposits to maximize eq. (16) subject to eq. (1). Let u_{ct} denote the marginal utility of consumption and $\Lambda_{t,t+1}$ the household's stochastic discount factor; then, a household's first order conditions for labor supply and consumption/saving decisions are

$$\mathbb{E}_t u_{ct} w_t = \zeta_0 H_t^{\zeta}\left(C_t - \mathcal{H}C_{t-1} - \frac{\zeta_0}{1+\zeta}H_t^{1+\zeta}\right)^{-\gamma} \tag{18}$$

$$1 = \mathbb{E}_t\left[R_{t+1}\Lambda_{t,t+1}\right] \tag{19}$$

$$D_t^{*,h} = \overline{D}^{*,h} + \frac{\mathbb{E}_t\left[\Lambda_{t,t+1}\left(\frac{e_{t+1}}{e_t}R_{t+1}^* - R_{t+1}\right)\right]}{\kappa_{D*}} \tag{20}$$

$$\mathcal{S}_t = \overline{\mathcal{S}} + \frac{\mathbb{E}_t\left[\Lambda_{t,t+1}\left(R_{t+1}^{knc} - R_{t+1}\right)\right]}{\kappa_S} \tag{21}$$

with

$$u_{ct} = \left(C_t - \mathcal{H}C_{t-1} - \frac{\zeta_0}{1+\zeta}H_t^{1+\zeta}\right)^{-\gamma} - \mathcal{H}\beta\mathbb{E}_t\left(C_{t+1} - \mathcal{H}C_t - \frac{\zeta_0}{1+\zeta}H_{t+1}^{1+\zeta}\right)^{-\gamma}$$

$$\Lambda_{t,t+1} = \beta\frac{u_{c,t+1}}{u_{ct}}$$

[11] For a complete examination of the labor supply function in the general case $\mathcal{H} \in [0, 1)$, see Appendix C.2.

The optimal demand for private securities and foreign currency-denominated bank deposits (eq. (20) and eq. (21), respectively) is increasing in the excess return of each asset but relative to the parameter that governs the marginal transaction cost. Notice that if the marginal transaction costs disappear (i.e. κ_{D*} and κ_S go to zero), households are able to engage in complete arbitrage and excess returns will tend to zero. On the contrary, when the marginal transaction costs are infinite, the demands for $D_t^{*,h}$ and $\mathcal{S}$ are completely unresponsive to excess returns and are given by $\overline{D}^{,h}$ and $\overline{\mathcal{S}}$, respectively.

Finally, when household's demand for bank deposits denominated in foreign currency differs from its frictionless level, endogenous deviations from the UIP condition emerge in equilibrium. Bear in mind, that a similar equation was obtained from banks' first order conditions whenever their incentive constraint binds. Therefore, when the incentive constraint for banks is binding and households are unable to engage in complete arbitrage, FX interventions are not neutral. However, if household's demand for bank deposits in foreign currency is infinitely responsive to arbitrage opportunities (i.e. transactions costs become increasingly smaller) the effect of FX interventions is completely neutralized.

2.4 The production sector

There are four types of non-financial firms making up the production side of the model economy: 1) non-commodity final good producers; 2) intermediate good producers; 3) capital good producers; and 4) the commodity production sector, which takes global commodity prices and external demand as given.

Non-Commodity Final Good Producers. Final goods in the non-commodity sector are produced under perfect competition and using a variety of differentiated intermediate goods y_{jt}^{nc}, with $j \in [0, 1]$, according to the following constant returns to scale technology

$$Y_t^{nc} = \left(\int_0^1 y_{jt}^{nc\,\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}} \tag{22}$$

where $\eta > 1$ is the elasticity of substitution across goods. The representative firm chooses y_{jt}^{nc} to maximize profits subject to the production function eq. (22) with profits given by:

$$P_t^{nc} Y_t^{nc} - \int_0^1 p_{jt}^{nc} y_{jt}^{nc} dj,$$

The first-order conditions for the jth input are

$$y_{jt}^{nc} = \left(\frac{p_{jt}^{nc}}{P_t^{nc}} \right)^{-\eta} Y_t^{nc}$$

$$P_t^{nc} = \left(\int_0^1 p_{jt}^{nc\,1-\eta} dj \right)^{\frac{1}{1-\eta}}$$

The final homogeneous good can be used either for consumption or to produce capital goods. In addition, part of the final good production is exported for foreign consumption.

Intermediate Good Producers. There is a continuum of monopolistically competitive firms, indexed by $j \in (0, 1)$, producing differentiated intermediate goods that are sold to final good producers. Each firm manufactures a single variety, face nominal rigidities in the form of price adjustment costs as in Rotemberg (1982) and pay for their capital expenditures in advance of production with funds borrowed from banks. Each intermediate good producer operates the following constant return to scale technology with three inputs: capital k^{nc}_{t-1}, imported goods m_t, and labor l_t

$$y^{nc}_{jt} = A^{nc}_t \left(\frac{k^{nc}_{j,t-1}}{\alpha_k}\right)^{\alpha_k} \left(\frac{m_{jt}}{\alpha_m}\right)^{\alpha_m} \left(\frac{h_{jt}}{1-\alpha_k-\alpha_m}\right)^{1-\alpha_k-\alpha_m} \tag{23}$$

where α_k, α_m, and $\alpha_k+\alpha_m \in (0,1)$. Also, A^{nc}_t denotes a neutral technology process common to all intermediate good producers that follows

$$\ln A^{nc}_t = (1-\rho_{A^{nc}}) \ln A^{nc} + \rho_{A^{nc}} \ln A^{nc}_{t-1} + u^{A^{nc}}_t \tag{24}$$

We assume that intermediate good producers issue equity, $\mathcal{S}_{j,t}$, to domestic households and borrow from banks in order to acquire capital for production. After obtaining funds, each intermediate good producer buys capital from capital good producers at a unitary price q^{nc}_t. Furthermore, in order to reflect the presence of credit dollarization in some EMEs and the fact that partially dollarized economies might be more vulnerable to external shocks, we assume that an intermediate good producer needs a combination of domestic and foreign currency-denominated loans to buy capital. The combination of both types of loans is achieved assuming a Cobb-Douglas technology that yields a unit measure of disposable funds, $\mathcal{F}_{j,t}$ or loan services. Thus, the loan bundle that an intermediate good producer needs to the buy capital good is the following:

$$\mathcal{F}_{j,t} = A^e l^{1-\delta f}_{j,t} \left(e_t l^*_{j,t}\right)^{\delta f} \tag{25}$$

where A^e is the productivity level for aggregate loan services, $l_{j,t}$ and $l*_{j,t}$ denote domestic and foreign currency-denominated bank loans respectively and the parameter δ^f controls for the degree of credit dollarization in the economy. Finally, at the end of the period, intermediate good producers sell the undepreciated capital, $\lambda_{nc} k^{nc}_{j,t-1}$, to capital good producers.

First-order conditions for intermediate good producers are presented in three groups[12], each associated with the following production stages: (i) cost minimization, (ii) borrowing from banks and issuing equity to households, and (iii) price setting. . The cost minimization stage yields the standard conditional demands for each input:

$$z_t = \alpha_k mc_t \frac{y^{nc}_{jt}}{k^{nc}_{j,t-1}} \tag{26}$$

$$e_t = \alpha_m mc_t \frac{y^{nc}_{jt}}{m_{j,t}} \tag{27}$$

$$mc_t = \frac{1}{A^{nc}_t} z^{\alpha_k}_t e^{\alpha_m}_t w^{1-\alpha_k-\alpha_m}_t \tag{28}$$

The borrowing stage is characterized by a non-arbitrage condition that defines the return on capital (see eq. (29) below) and real loan demands in domestic and foreign currency (eq. (30) and eq. (31)):

$$R^k_t = \frac{z_t + \lambda_{nc} q^{nc}_t}{q^{nc}_{t-1}} \tag{29}$$

$$l_{j,t} = (1-\delta^f) \left(\frac{\mathbb{E}_t \Lambda_{t,t+1} R^k_{t+1}}{\mathbb{E}_t \Lambda_{t,t+1} R^l_{t+1}}\right) \mathcal{F}_{j,t} \tag{30}$$

[12]See appendix C.3 for a detail derivation of the following equations.

$$e_t l^*_{j,t} = \delta^f \left(\frac{\mathbb{E}_t \Lambda_{t,t+1} R^k_{t+1}}{\mathbb{E}_t \Lambda_{t,t+1} \frac{e_{t+1}}{e_t} R^{l*}_{t+1}} \right) \mathcal{F}_{jt} \tag{31}$$

$$q^{nc}_t k^{nc}_{j,t} = \mathcal{S}_{j,t} + \mathcal{F}_{j,t} \tag{32}$$

In equilibrium, issuing equity and borrowing from banks are considered to be perfect substitutes to intermediate good producers, since both, generate equal expected real costs. The demand schedules for domestic and foreign currency loans depend directly on the expected return on capital as well as on the current value of acquired capital by each firm and inversely on the expected interest rate cost of each type of credit. Therefore, in equilibrium the degree of credit dollarization, given by $\frac{eL^*_t}{L_t + eL^*_t}$ where e is the steady-state real exchange rate, is an endogenous variable that depends on domestic financial conditions. The parameter δ^f determines if intermediate good producers need to borrow in foreign currency from banks. Whenever $\delta^f = 0$, the demand for foreign currency loans is zero and banks' balance sheet is such that there is no asset dollarization (see Section 4.4).

Finally, the price setting stage is characterized by the following New Keynesian Phillips curve:

$$(1 + \pi_t)\pi_t = \frac{1}{\kappa}(1 - \eta + \eta mc_t) + \mathbb{E}_t \left[\Lambda_{t,t+1}(1 + \pi_{t+1})\pi_{t+1} \frac{Y^{nc}_{t+1}}{Y^{nc}_t} \right] \tag{33}$$

Capital Good Producers. There is a continuum of capital producers operating in a competitive market. Each capital good producer uses final goods as inputs in the form of non-commodity investments, as well as the undepreciated capital bought from intermediate good producers. New capital is produced using the following technology:

$$K^{nc}_t = I^{nc}_t + \lambda_{nc} K^{nc}_{t-1} \tag{34}$$

where K^{nc}_t is sold to intermediate good producers at the price q^{nc}_t. Producing capital implies an additional cost of $\Phi^{nc}\left(\frac{I^{nc}_t}{I^{nc}}\right) I^{nc}_t$, which represents the adjustment cost of investment. The latter assumption is introduced to replicate some empirical moments [13]. Given that households own the capital good firm, the objective of a capital producer is to choose $\{I^{nc}_{t+j}\}_{j\geq 0}$ to solve:

$$\mathbb{E}_t \left[\sum_{j=0}^{\infty} \Lambda_{t,t+j} \left(q^{nc}_{t+j} I^{nc}_{t+j} - \left[1 + \Phi^{nc}\left(\frac{I^{nc}_{t+j}}{I^{nc}} \right) \right] I^{nc}_{t+j} \right) \right]$$

Profit maximization implies that the price of capital goods is equal to the marginal cost of investment good production as follows:

$$q^{nc}_t = 1 + \Phi^{nc}\left(\frac{I^{nc}_t}{I^{nc}}\right) + \left(\frac{I^{nc}_t}{I^{nc}}\right) \partial\Phi^{nc}_t \tag{35}$$

where $\partial\Phi^{nc}_t$ denotes the derivative of $\Phi^{nc}(.)$ evaluated at $\frac{I^{nc}_t}{I^{nc}}$.

Commodity Sector. Commodity price movements play a major role in commodity-exporting EMEs. Conventional wisdom suggests that terms-of-trade fluctuations constitute an important driver of business cycle fluctuations in EMEs. In particular, commodity booms generate real as well as credit booms.[14]

[13] The function $\Phi^{nc}()$ must satisfy the following restrictions: $\Phi^{nc}(1) = (\Phi^{nc})'(1) = 0$ and $(\Phi^{nc})''(.) > 0$.

[14] For empirical evidence on this fact, see Fornero et al. (2015), Shousha (2016), Fernández et al. (2017), Garcia-Cicco et al. (2017), and Drechsel and Tenreyro (2018).

We introduce a commodity sector with a representative firm that produces a homogeneous commodity good taking global commodity prices and external demand as given. We assume this firm is owned by both foreign and domestic agents. Commodity production is entirely exported abroad and is conducted using capital specific to this sector as the only input. Capital is acquired directly from final good producers and is used to produce commodity-sector capital without any lending from the banking system. Technology in this sector is

$$Y_t^c = A^c (K_{t-1}^c)^{\alpha_c} \tag{36}$$

where Y_t^c is the commodity production, K_t^c is the specific capital for the commodity sector, and A^c is the productivity level in this sector. We assume that the commodity firm's ownership is divided between domestic and foreign shareholders. Specifically, domestic households own a fraction χ^c of the total firm's value while foreign families own $(1-\chi^c)$. Moreover, we assume that commodity firm's should pay a fraction τ^c of its profits as domestic government taxes.

The representative commodity producer faces investment adjustment costs of $\Phi^c\left(\frac{I_t^c}{I^c}\right)$. Thus, capital accumulation is done through the following equation:

$$K_t^c = I_t^c + \lambda_c K_{t-1}^c \tag{37}$$

The representative producer problem in the commodity sector is to choose $\{K_{t+s}^c\}_{s\geq 0}$ and $\{I_{t+s}^c\}_{s\geq 0}$ to maximize[15]

$$\sum_{s=0}^{\infty} \Lambda_{t,t+s}(1-\tau^c)\left(p_{t+s}^c A^c (K_{t+s-1}^c)^{\alpha_c} - \left[1 + \Phi^c\left(\frac{I_{t+s}^c}{I^c}\right)\right] I_{t+s}^c\right)$$

subject to eq. (36). The first-order conditions for the above problem are

$$q_t^c = 1 + \Phi^c\left(\frac{I_t^c}{I^c}\right) + \left(\frac{I_t^c}{I^c}\right)\partial\Phi_t^c \tag{38}$$

$$1 = \mathbb{E}_t\left[\Lambda_{t,t+1} R_{t+1}^{kc}\right] \tag{39}$$

$$R_t^{kc} = \frac{\alpha_c p_t^c \frac{Y_t^c}{K_{t-1}^c} + q_t^c \lambda^c}{q_{t-1}^c} \tag{40}$$

where $\partial\Phi_t^c$ denotes the derivative of $\Phi^c(.)$ evaluated at $\frac{I_t^c}{I^c}$ and $(1-\tau^c)q_t^c$ is the shadow price for the commodity-specific stock of capital. We assume that the domestic household owns a higher fraction of the representative commodity producer. Therefore, the stochastic discount factor used by the commodity producer is also the one used by domestic households.

Finally, we assume that a fraction $(1-\chi^c)$ of the profits is transferred abroad to foreign owners. The aggregate profit in the commodity sector is given by

$$\Pi_t^c = p_t^c A^c (K_{t-1}^c)^{\alpha_c} - \left[1 + \Phi^c\left(\frac{I_t^c}{I^c}\right)\right] I_t^c \tag{41}$$

It is worth mentioning that in our framework a commodity boom directly raises the demand for domestic final goods, since non-commodity investment is used as input to produce specific capital for the commodity sector. The latter occurs independently of the standard wealth effect that surges in commodity prices generate when this sector is modeled as an exogenous endowment. Furthermore, the demand for credit also increases as a response to both, the wealth effect and the increase in the production of intermediate goods needed to support the higher demand for final goods.

[15] We assume that foreign stochastic discount factor is the same of the their domestic counterpart. Hence, we use $\Lambda_{t,t+1}$ as the discount factor for future commodity sector's cash-flows independent of its ownership.

2.5 External Sector

We assume that foreign demand for non-commodity final goods is a decreasing function of the relative price $\frac{1}{e_t}$ but increasing with the foreign income Y_t^* as

$$Y_t^{nc,x} = e_t^{\varphi} Y_t^* \tag{42}$$

where $\varphi > 0$ is the price elasticity.

The foreign sector block hast its own dynamic outside the domestic macroeconomic equilibrium and does not have feedback from domestic variables. We consider as external variables foreign output Y_t^*, foreign interest rate R_t^*, and the commodity price index p_t^{wc}; and collect these variables in vector $\hat{\mathbf{X}}_t$, which captures the cyclical movements of these variables in an SVAR block; i.e.,

$$\hat{\mathbf{X}}_t = \begin{bmatrix} \hat{Y}_t^* \\ \hat{R}_t^* \\ \hat{p}_t^* \end{bmatrix}$$

where $\hat{Y}_t^* = \ln \frac{Y_t^*}{Y^*}$, $\hat{R}_t^* = R_t^* - R^*$, and $\hat{p}_t^* = \ln \frac{p_t^{wc}}{p^{wc}}$. Then, we assume that $\hat{\mathbf{X}}_t$ follows a vector autoregressive equation written as

$$\hat{\mathbf{X}}_t = \mathbf{C}\hat{\mathbf{X}}_{t-1} + \mathbf{B}\mathbf{u}_t^X \tag{43}$$

where $\mathbf{C}$ and $\mathbf{B}$ are 3×3 matrices that rule the dynamics of the vector $\hat{\mathbf{X}}_t$, and $\mathbf{u}_t^X$ is the vector of external structural shocks from which we analyze its consequences. Section 3 presents further details in the way we estimate eq. (43) and identify its structural shocks.

2.6 Central Government

The consolidated government collects taxes from households and receives a fraction χ^c of commodity firms' profits. These resources are then used to finance public consumption G_t and central bank operational losses CB_t:

$$\tau^c \Pi_t^c + T_t = CB_t + G_t \tag{44}$$

where G_t is modeled as a first order autoregressive process written as

$$G_t = (1 - \rho_G)G + \rho_G G_{t-1} + u_t^G \tag{45}$$

where ρ_G controls the persistence of public expenditure dynamics. It is worthy noticing that eq. (44) indicates that either commodity price cycles or central bank operational losses will strongly affect household's decisions through variations in lump-sum taxes.

We also assume that the monetary authority sets the short-term nominal interest rate i_t according to a simple feedback rule following a standard Taylor-type rule:

$$i_t - i = \rho_i(i_{t-1} - i) + (1 - \rho_i)\left[\omega_\pi \pi_t + \omega_y \ln\left(\frac{\text{GDP}_t}{\text{GDP}}\right)\right] + u_t^i \tag{46}$$

where ρ_i measures the persistence of the policy rate, ω_π controls the degree of the policy rate response to inflation variations, and u_t^i represents monetary policy shocks. In order to converge to a stable equilibrium, this rule should satisfy the Taylor principle; i.e., $\omega_\pi > 1$.

2.7 Market Equilibrium

The non-commodity output is either consumed, invested, exported, or used to pay the cost of adjusting prices, the cost of changing investment decisions, and the resources wasted after aggregating funds at the intermediate good producer level,

$$Y_t^{nc} = C_t + G_t + I_t^{nc} + I_t^c + Y_t^{x,nc} + \text{REST}_t \tag{47}$$

where

$$\text{REST}_t = \frac{\kappa}{2}\pi_t^2 Y_t^{nc} + e_t \frac{\kappa_{D*}}{2}\left(D_t^{*,h} - \overline{D}^{*,h}\right)^2 + \frac{\kappa_S}{2}\left(\mathcal{S}_t - \overline{\mathcal{S}}\right)^2 + \Phi^c\left(\frac{I_t^c}{I^c}\right) + \Phi^c\left(\frac{I_t^c}{I^c}\right) + L_t + e_t L_t^* - \mathcal{F}_t$$

We should impose a market clearing condition also for the foreign currency deposits:

$$D_t^* = D_t^{*,h} + D_t^{*,f} \tag{48}$$

Gross Domestic Product (GDP) is the aggregate value added of the non-commodity and commodity sectors, all priced at constant prices:

$$\text{GDP}_t = Y_t^{nc} - eM_t + p^c Y_t^c \tag{49}$$

where p^c and e are the steady-state levels for the commodity price index and the real exchange rate, respectively. Therefore, GDP_t captures only real output movements and is not affected by valuation effects.

The aggregate net foreign asset position NFAP_t, which is equal to FX official reserves minus aggregate foreign liabilities in the baking system (i.e. $F_t - D_t^{*,f}$), evolves through the trade balance net of the fraction of commodity firms' profits transferred abroad and the financial income of net foreign assets from the previous period,

$$e_t\left[\text{NFAP}_t - R_t^*\text{NFAP}_{t-1}\right] = Y_t^{x,nc} + p_t^c Y_t^c - e_t M_t - (1-\tau^c)(1-\chi^c)\Pi_t^c \tag{50}$$

Finally, since optimal banks' decisions do not depend on bank-specific factors, the aggregation for the banking system variables is straightforward. In appendix C.1, we show that the total net worth evolves according to:

$$N_t = (\sigma + \xi)\left(R_t^l L_{t-1} + R_t^{l*} e_t L_{t-1}^* + R_t^b B_{t-1}\right) - \sigma R_t D_{t-1} - \sigma e_t R_t^* D_{t-1}^* \tag{51}$$

3 Parametrization Strategy

We discipline the model to replicate some relevant unconditional and conditional moments for the Peruvian economy. We calibrate a subset of the parameters to be consistent with some steady state targets associated with historical means. Additionally, we follow Schmitt-Grohe and Uribe (2018) to estimate another subset of parameters by using a limited information method based on an impulse response matching function estimator. For this purpose, we estimate an SVAR with two recursive blocks. Then, we estimate some parameters of our macroeconomic model by minimizing the distance between the structural impulse responses implied by the macroeconomic model and the corresponding empirical impulse responses implied by the SVAR model. Let Ξ be the subset of parameters to be estimated by matching the impulse responses to external shocks, $\mathcal{M}^{\text{data}}$ the corresponding empirical impulse responses from the SVAR model, and $\mathcal{M}^{\text{model}}$ the theoretical counterpart of $\mathcal{M}^{\text{data}}$. Then we set Ξ to be the solution to the following problem

$$\Xi^{*} = arg\min_{\Xi} \quad \sum_{i=1}^{k} \frac{1}{\varrho_i} \times \left[\mathcal{M}_i^{\text{model}}(\Xi) - \mathcal{M}_i^{\text{data}}\right]^2 \tag{52}$$

where ϱ_i denotes the width of the 68% confidence interval associated with the ith variable in $\mathcal{M}^{\text{data}}$.

Empirical VAR Specification. We consider an SVAR model with two blocks similar to Canova (2005), Cushman and Zha (1997), and Zha (1999). Let $\mathbf{X}_t$ denote the vector of foreign variables and $\mathbf{D}_t$ the vector of domestic variables. In the baseline specification, each block is composed by the following variables:

$$\mathbf{X}_t = \begin{bmatrix} Y_t^* \\ R_t^* \\ p_t^{wc} \end{bmatrix}, \qquad \mathbf{D}_t = \begin{bmatrix} tb_t \\ GDP_t \\ C_t \\ I_t \\ L_t \\ eL_t^* \\ e_t \end{bmatrix}$$

The external variables Y_t^*, R_t^*, and p_t^{wc} denote the real GDP index for the G-20 group of countries, the Baa U.S corporate spread, and a metal export price index relevant for Peru. The domestic variables GDP_t, C_t, I_t, L_t, and $e_tL_t^*$ denote real indexes for Peru's GDP, consumption, investment, and real bank lending in domestic currency as well as in foreign currency respectively, while e_t denotes the bilateral real exchange rate and tb_t the trade balance-to-GDP ratio. Following Canova (2005), the baseline specification considers $\mathbf{X}_t$ as an exogenous block, with no feedback dynamics from the domestic block, $\mathbf{D}_t$, at any point in time. Therefore, like much of the related literature, the main identification assumption is that an emerging small open economy as Peru, takes as given world prices and quantities. The baseline specification assumes that all variables are expressed in log-levels. The only variables expressed in percentage terms are R_t^* and tb_t. Therefore, we consider an SVAR in levels with zero restrictions between blocks and a linear or quadratic time trend in order to capture the SOE assumption of the Peruvian economy, as well as to control for time trends. It is important to mention that shocks within each block are identified recursively with zero contemporaneous restrictions.

Formally, consider the following restricted block VAR model with deterministic trend:

$$\begin{bmatrix} \mathbf{X}_t \\ \mathbf{D}_t \end{bmatrix} = \begin{bmatrix} \Phi_X \\ \Phi_D \end{bmatrix} G(t) + \begin{bmatrix} \Phi_{XX}^1(L) & \mathbf{0} \\ \Phi_{DX}^1(L) & \Phi_{DD}^1(L) \end{bmatrix} \begin{bmatrix} \mathbf{X}_{t-1} \\ \mathbf{D}_{t-1} \end{bmatrix} + \begin{bmatrix} \mathbf{v}_t^X \\ \mathbf{v}_t^D \end{bmatrix} \tag{53}$$

where $G(t)$ measures a deterministic time trend[16]. Φ_X, Φ_D are vectors of ones, $\mathbf{v}_t^X \sim N(0, \Sigma_{\mathbf{v}^F})$ and $\mathbf{v}_t^D \sim N(0, \Sigma_{\mathbf{v}^D})$. Hence, the underlying SVAR model is

$$\begin{aligned} \begin{bmatrix} \Theta_{XX}^0 & \mathbf{0} \\ \Theta_{DX}^0 & \Theta_{DD}^0 \end{bmatrix} \begin{bmatrix} \mathbf{X}_t \\ \mathbf{D}_t \end{bmatrix} &= \begin{bmatrix} \Theta_X \\ \Theta_D \end{bmatrix} G(t) \\ &+ \begin{bmatrix} \Theta_{XX}^1(L) & \mathbf{0} \\ \Theta_{DX}^1(L) & \Theta_{DD}^1(L) \end{bmatrix} \begin{bmatrix} \mathbf{X}_{t-1} \\ \mathbf{D}_{t-1} \end{bmatrix} + \begin{bmatrix} \mathbf{u}_t^X \\ \mathbf{u}_t^D \end{bmatrix} \end{aligned} \tag{54}$$

[16] Like the SVAR model, the DSGE model considers deterministic time trends that are removed before the matching procedure.

The data has a quarterly frequency and covers from 2002Q1 to 2017Q2 for the domestic block and from 1980Q1 to 2017Q2 for the foreign block. Following Fernández et al. (2017), we first estimate the foreign block separately and impose the corresponding estimated parameters in the estimation of the domestic block.

Calibration based on previous literature. Some parameters are parametrized at standard values and for some others the parametrization is based on previous works. The elasticity of intertemporal substitution for household preferences is equal to 1/2 (i.e., $\gamma = 2$). Consistent with Céspedes and Rendón (2012), households preferences have a Frisch elasticity of the labor supply equal to 1/3 (i.e., $\zeta = 3$).

Concerning the productive sector, the elasticity of substitution among intermediate goods is set at 6 and the capital depreciation rate is set at 10% annually for both sectors. We also assume that foreign agents have the ownership of commodity firms (i.e., $\chi^c = 0$) but there is a commodity profit tax of 60% which is in line with Garcia-Cicco et al. (2017). Moreover, the parameters controlling the conventional monetary policy response (ρ_i; ω_π; ω_y) are parametrized using previous work (see Castillo et al. (2009) and Winkelried (2013)). Finally, and based on ABK's steady-state analysis, we fix the fraction of total assets transferred to start-up bankers, ξ, in a tiny and positive level, $1e^{-10}$. Table 2 summarizes this raw parametrization.

Table 2. Raw Parametrization

Description	Parameter	Value
Elasticity of Intertemporal Substitution	γ	2.00
Inverse Frisch Elasticity	ζ	3.00
Elasticity of Substitution of Goods	η	6.00
Undepreciated NC Capital Rate	λ^{nc}	0.975
Undepreciated C Capital Rate	λ^{c}	0.975
Domestic Ownership on Commodity Firms	χ^{c}	0.00
Tax on Commodity Sector Profit	τ^{c}	0.60
Banker's Start-Up Transfers	ξ	1.00e-10
MP Rate Smoothing	ρ_i	0.70
MP Rate response to Inflation	ω_π	1.50
MP Rate response to Output Gap	ω_y	0.125

Steady-State Targets. Regarding the banking side of the model, we parametrize (ϖ^*, ϖ^b, ξ, θ, $\varkappa$, δ^f) to be consistent with the following steady-state financial targets: annual domestic currency loan return of 6%, annual foreign currency loan return of 4%, annual central bank bond return of 4%, domestic currency leverage of 3.50, dollar deposits to total assets ratio of 53.5%, and credit dollarization rate of 42.5%.

Concerning the non-banking sector, the vector of parameters (A^e, ζ_0, Y^*, p^{wc}, A^{nc}) is calibrated to attain 8% of annual non-commodity capital return, 0.33 of worked hours, and real exchange rate, commodity price, and GDP set at 1 in the steady state. Furthermore, ($\overline{D}^{*,h}$, $\overline{\mathcal{S}}$, α^c, A^c, α^k, α^m, B^{fx}) is parametrized to set the main sectoral, demand-side, and stock ratios equal to their empirical counterpart. Table 7 in Appendix A summarizes our targeted parametrization strategy described above.

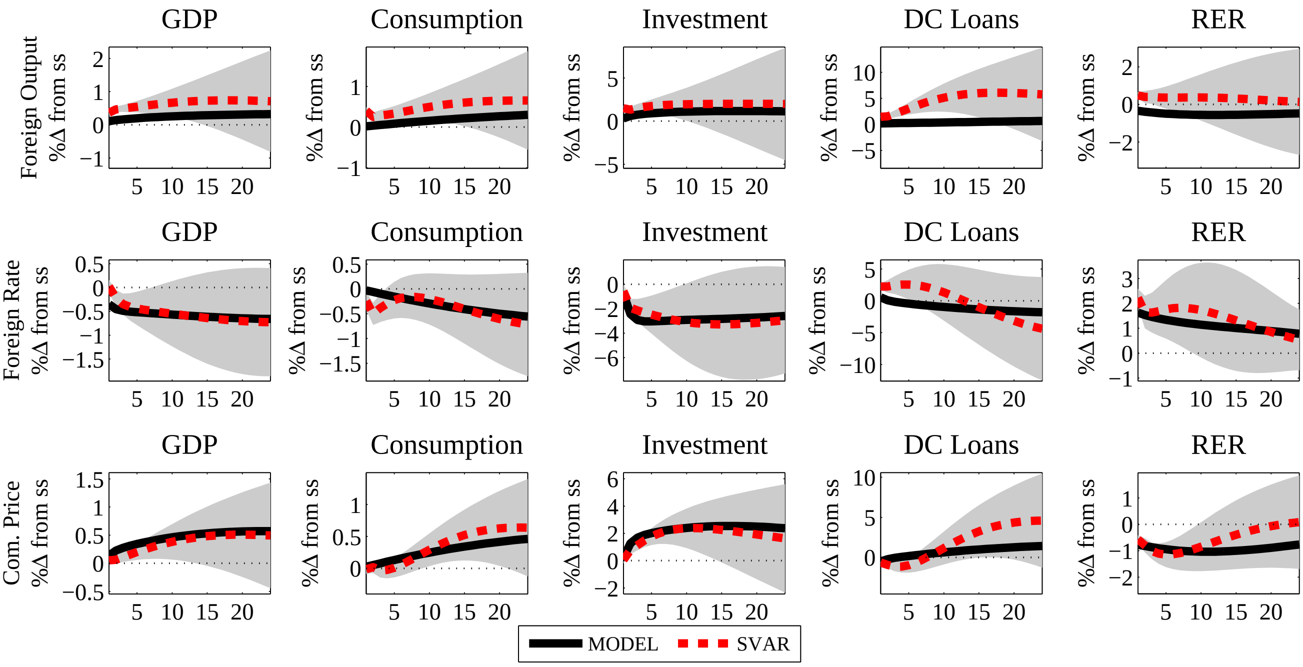
Impulse Response Matching. The rest of the parameters are estimated to match impulses responses to external shocks between the SVAR and the DSGE model. We use the responses of GDP, consumption, investment, DC loans, and real exchange rate for the first 24 periods in order compute the IRF Matching.

Our estimation results are summarized in Table 3. Furthermore, Fig. 2 compares the corresponding impulse-responses associated with the empirical and structural models. Our empirical model indicates that a foreign interest rate shock cause a real exchange rate depreciation and a contractionary effect on credit and output. On the other hand, global demand and commodity price shocks are expansionary in terms of domestic output, investment, and credit. These empirical responses are very closely followed by the theoretical responses of our DSGE model.

Table 3. IRF Matching Parametrization

Description	**Parameter**	**Value**
Non-Commodity Capital Adjustment Cost	κ_{Inc}	0.05
Commodity Capital Adjustment Cost	κ_{Ic}	1.20
FX Intervention response to RER	υ_e	9.71
Non-Commodity Exports Price Elasticity	φ	1.49e-05
Household FC Deposit Adjustment Cost	κ_{D*}	17.91
Houosehold Capital Adjustment Cost	κ_S	0.01
Household Habit Formation	$\mathcal{H}$	0.98

Figure 2. Impulse Response Matching



Note. Solid black (red dash) lines show point estimates of impulse response of the DSGE model (SVAR model); and 68% confidence bands associated with the SVAR's impulse response are depicted with dark-gray shaded areas.

4 Numerical Experiments

In this section, we perform several simulations designed to analyze how FX interventions affect the response of the model economy to external shocks. Specifically, we focus on the transmission of a sudden increase in the foreign interest rate and a global commodity boom.[17] The foreign block in the DSGE model is calibrated as in the estimated SVAR. However. we isolate the correlation within this block and focus our analysis in the effect of each external shock as the sole external mechanism.

We begin by analyzing the responses of aggregate variables to external shocks under different exchange rate regimes: flexible exchange rate regime vs. FX intervention regime. Under the FX intervention regime, the central bank "leans against the wind" with respect to real exchange rate fluctuations by implementing eq. (15), but its interest rate rule is also active. Next, we simulate an exogenous, sufficiently large unanticipated and permanent accumulation (purchase) of FX reserves and study its transmission mechanism. Finally, we explore the way our main results change after the relaxation of some assumptions in our baseline framework.

Recent empirical literature about FX interventions (e.g., see Fratzscher et al. (2019)) uses distinct criterias to measure the effectiveness of this policy instrument. The evaluation of these criterias is related to the objective of our numerical experiments. For instance, the impulse-response analysis can be associated to the the *event criterion* which tests whether the exchange rate moves in the intendend direction during the intervention episode (e.g., if the central bank buys foreign currency, the real exchange rate should depreciates). Moreover, according to the smoothing criterion, we evaluate whether FX interventions limit the real exchange rate volatility (see Tab. 4). Although this literature has studied the effects of FX intervention over the exchange rate, in this paper we extend the usage of these criterias to analyse the FX effectiveness over other macroeconomic variables. Furthermore, our general equilibrium framework is also useful to explore the effectiveness of FX interventions in terms of welfare and we do it in Section 5.

Quantitatively our results suggest that under the FX intervention (FXI) regime the macroeconomic volatility is reduced relative to the flexible exchange rate (FER) regime, reflecting that FX interventions play the role of an external shock absorber. As expected, the volatility of the real exchange rate (RER) is reduced by 68%, while the corresponding volatility of total credit is reduced by 82% (see Table 4). Simultaneously, the volatility of output, investment, and consumption falls by around 70, 65, and 7 percent, respectively. Hence, the volatility of inflation diminishes by 58 percent under the FXI regime.

[17]In Appendix B.3, we also show the responses to an increase in global GDP. Additionally, in Appendix B.4, we present responses to external shocks when the central bank FX intervention rule respond to real depreciation or to the interest rate spread between foreign and domestic interest rates instead of responding to real exchange rate deviations. We use the following FX intervention rule:

$$\ln B_t = (1-\rho_B)\ln B + \rho \ln B_{t-1} - \upsilon_e(\ln e_t - \ln e) - \upsilon_{\Delta e}\Delta e_t - \upsilon_{\text{spread}}\left[\left(\frac{e_t}{e_{t-1}}R_t^* - R_t\right) - (R^* - R)\right]$$

We obtain very similar results with both FX intervention rules under a proper calibration of these different types of FX rule. But we suggest that an FX intervention rule in terms of interest rate spreads is not as implementable as an FX intervention rule that responds to exchange rate deviations from steady-state.

Table 4. Unconditional Volatilities

	FXI (a)	FER (b)	(a)/(b)
RER	2.36	7.32	0.32
Real Depreciation	1.79	6.76	0.26
GDP	0.68	2.24	0.30
Investment	4.21	11.93	0.35
Consumption	0.23	0.25	0.93
Total Credit	1.25	6.79	0.18
Inflation	0.29	0.70	0.42
Currency Mismatch	2.11	6.07	0.35

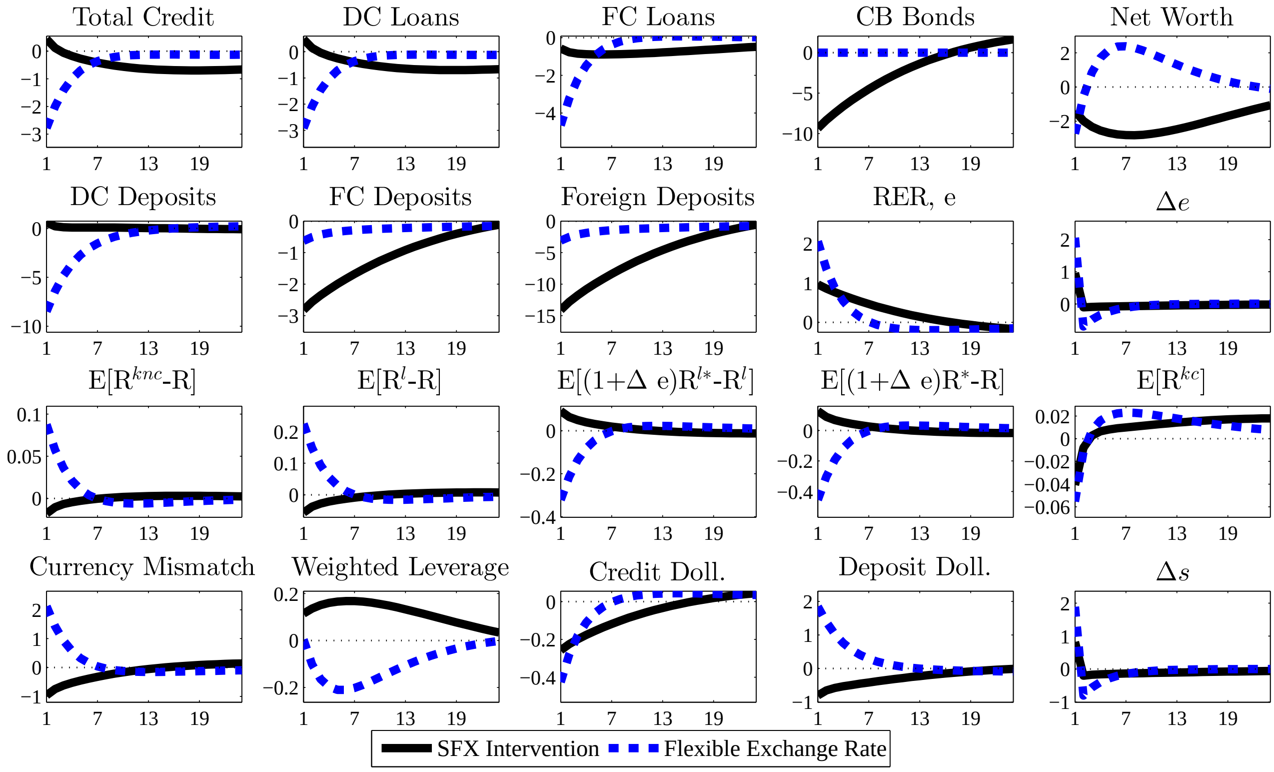
Note. "FXI" is the abbreviation for *Foreign Exchange Intervention* regimen and "FER" for the *Flexible Exchange Rate* regimen. The computation only considers external shock volatilities and is based on 1500 replications of 120 periods simulated trajectories.

In the following numerical experiments, we discuss the mechanisms through which FX interventions stabilize the macroeconomic variable responses in the presence of external shocks. Furthermore, we define two measures of financial dollarization for each side of banks' balance sheet: Credit dollarization $\frac{eL_t^*}{L_t+eL_t^*}$ is defined to be total foreign currency loans as a fraction of total lending and it is evaluated at the steady-state real exchange rate. Deposit dollarization is defined as banks' foreign borrowing as a fraction of total banks' liabilities evaluated also at the steady-state real exchange rate; i.e., $\frac{eD_t^*}{D_t+eD_t^*}$.

4.1 Foreign Interest Rate Shock

Figure 3 and Figure 4 show responses to an unexpected increase of 20 basis points in the foreign interest rate of financial and macroeconomic variables respectively. The dotted line reports responses under flexible exchange rate (i.e., $\upsilon_e = 0$) while the solid line represents the economy under the FX intervention policy. We first describe the transmission mechanism under exchange rate flexibility.

Figure 3. Responses of financial variables to a foreign interest rate shock



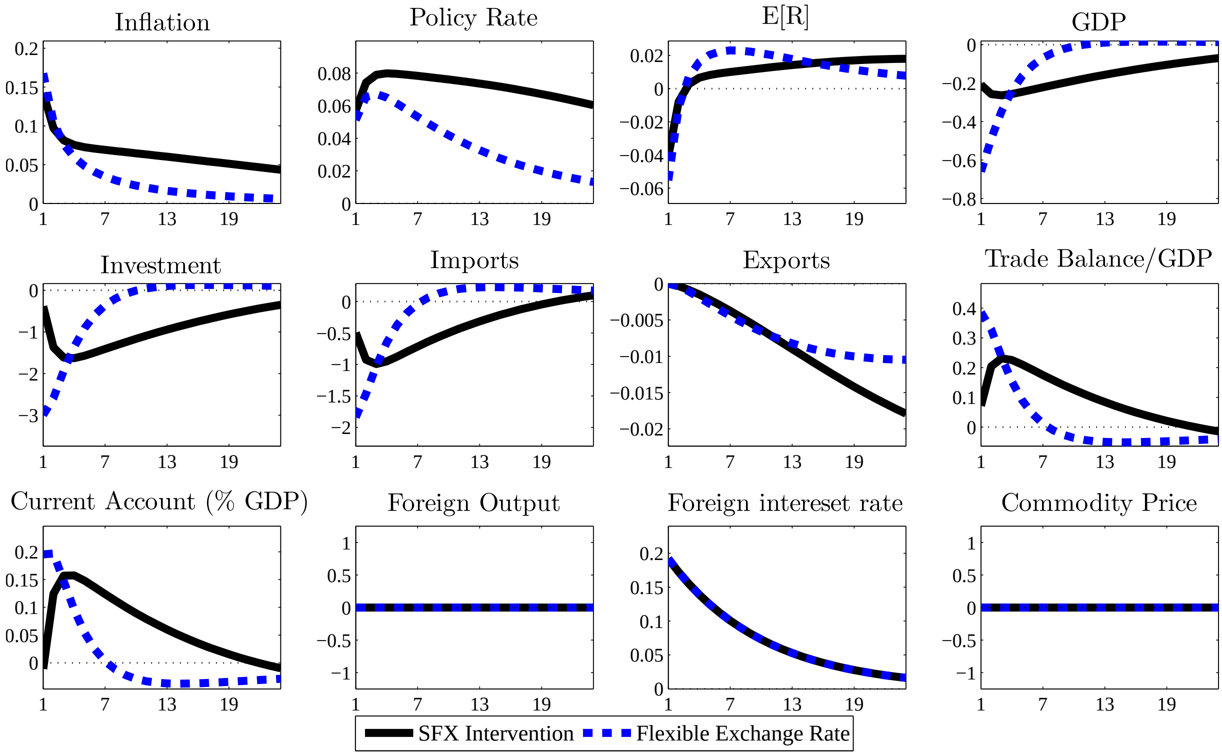
Note. The response of each quantity or index variable is presented as percent deviation from its steady-state, while the response of any rate variable is displayed in percentage-point deviations from its steady-state. e_t, Δe, and Δs denotes the real exchange rate, real depreciation, and nominal depreciation, respectively. $\mathbb{E}_t\left[\frac{e_{t+1}}{e_t}R^{l*}_{t+1} - R^l_{t+1}\right]$ and $\mathbb{E}_t\left[\frac{e_{t+1}}{e_t}R^{*}_{t+1} - R_{t+1}\right]$ measure the relative return of lending in foreign currency as well as the relative cost of borrowing in foreign currency from the point of view of banks.

Initially, the real exchange rate (RER in Fig. 3) depreciates by 2.1% and the economy experiences a contractionary financial effect. Since banks are exposed to currency mismatches in their balance sheets, the real exchange depreciation negatively affects banks' net worth and total credit; and ultimately generates a recession. Net worth declines at impact, but shows a fast recovery and then stabilizes around zero. Although the real exchange rate depreciates immediately after the shock, agents expect an exchange rate appreciation (see the dynamics of RER and real depreciation, Δe, in Fig. 3). The expected exchange rate appreciation modifies the relative costs and returns of borrowing and lending in foreign currency with respect to domestic currency, thereby changing the composition of banks' balance sheets. Thus, banks realize that borrowing in foreign currency is cheaper than in domestic deposits, and that lending in foreign currency becomes less profitable than lending in domestic currency. Consequently, banks reduce borrowing in both currencies but with a higher deposit dollarization (an impact of around 2 percentage points right after the shock occurs) and reduce lending in foreign currency implying a lower credit dollarization (an impact of -0.4 percentage points right after the shock).[18] Hence, under a flexible exchange rate regime,

[18] From the point of view of intermediate good producers, a foreign interest rate shock produces a substitution and an income effect with respect to their demand of each type of loans. The parametrization of the model, more precisely the IRF matching, is such that the income effect is much stronger than the substitution effect. Therefore, even though, the expected spread $\mathbb{E}_t\left[\frac{e_{t+1}}{e_t}R^{l*}_{t+1} - R^l_{t+1}\right]$ declines as a response

the exchange rate depreciation induced by the increase in the foreign interest rate raises the size of the currency mismatch, thereby reducing the intermediation capacity of banks: lending in both currencies declines by around 2.8%.

FIGURE 4. RESPONSES OF MACROECONOMIC VARIABLES TO A FOREIGN INTEREST RATE SHOCK



Note. The response of each quantity or index variable is presented as percent deviation from its steady-state, while the response of any rate variable is displayed in percentage-point deviations from its steady-state.

Financial conditions are reflected on interest rate spreads and macroeconomic variables. In particular, right after the foreign interest rate increases, the reduction of the bank's lending capacity is reflected in the increment of the expected interest rate spread of domestic currency lending relative to domestic currency borrowing as well as the expected excess return of non-commodity capital over domestic deposits in 0.2 p.p. (0.8 annual p.p.) and in 0.1 p.p. (0.4 annual p.p.), respectively. Therefore, investment falls by 3%, generating a persistent recession, with GDP falling by 0.7% (see Fig. 4). Finally, exchange rate depreciation raises inflation by 0.2% on impact, since the marginal cost of intermediate good producers depends on an imported input. The increase in inflation leads to a higher interest rate.

When the central bank responds to a foreign interest rate shock implementing FX interventions together with its standard monetary policy rule, both financial and macroeconomic variables are stabilized. The effect of FX interventions on the transmission mechanism of an external shock operates through two main channels: the exchange rate smoothing channel and the balance sheet substitution channel.

The Exchange Rate Smoothing Channel. When the incentive constraint binds, FX

to an increase in the foreign interest rate, intermediate good producers reduce borrowing in both currencies due to the contraction in aggregate lending and investment.

interventions modify the net asset foreign position of the economy, as well as the interest rate spread between foreign borrowing and domestic deposits. In particular, the central bank responds to an increase in the foreign interest rate by selling official FX reserves. Therefore, exchange rate dynamics change relative to the flexible exchange rate regime. At impact, the real exchange rate depreciates by 1% under the FX intervention regime, instead of 2.6% under the flexible exchange rate regime. After the impact, FX interventions successfully stabilize future real exchange rate appreciations.

As a result of smoothing the real exchange rate response, banks' net worth declines less at impact under the FX intervention regime (around 1% instead of 2.5% under the flexible exchange rate regime, see Fig. 3). The smoother pattern for the real exchange rate modifies the cost of borrowing in foreign currency relative to domestic currency deposits. In particular, under the FX intervention regime, the expected interest rate spread of domestic-currency borrowing over domestic-currency deposits raises around 0.1 percentage points instead of falling in 0.4 percentage points under free-floating exchange rate. Hence, contrary to the free-floating regime, deposit dollarization declines by one percentage point at impact. Similarly, the expected interest rate spread of foreign-currency loans over domestic currency loans is more stable, implying that credit dollarization falls but not as much as under exchange rate flexibility.

The Balance Sheet Substitution Channel. This channel is associated with central bank sterilization operations to keep domestic liquidity constant after FX sales. The central bank buys bonds that are in banks' balance sheets, ultimately affecting their size and composition. Consequently, this operation frees funds, which are used by banks to lend in both currencies. In this regard, FX interventions are similar to credit policy in the non-conventional monetary policy literature for closed economies.

Quantitatively, our results suggest that the sterilization leg of FX sales implies that central bank bonds in banks' balance sheets decline by 10% at impact (see the response of CB Bonds in Fig. 3). As a result, lending in both currencies decline less than under exchange rate flexibility. In particular, at through, total loans fall by 0.7% when FX interventions are used, instead of declining by 2.8% under free floating.

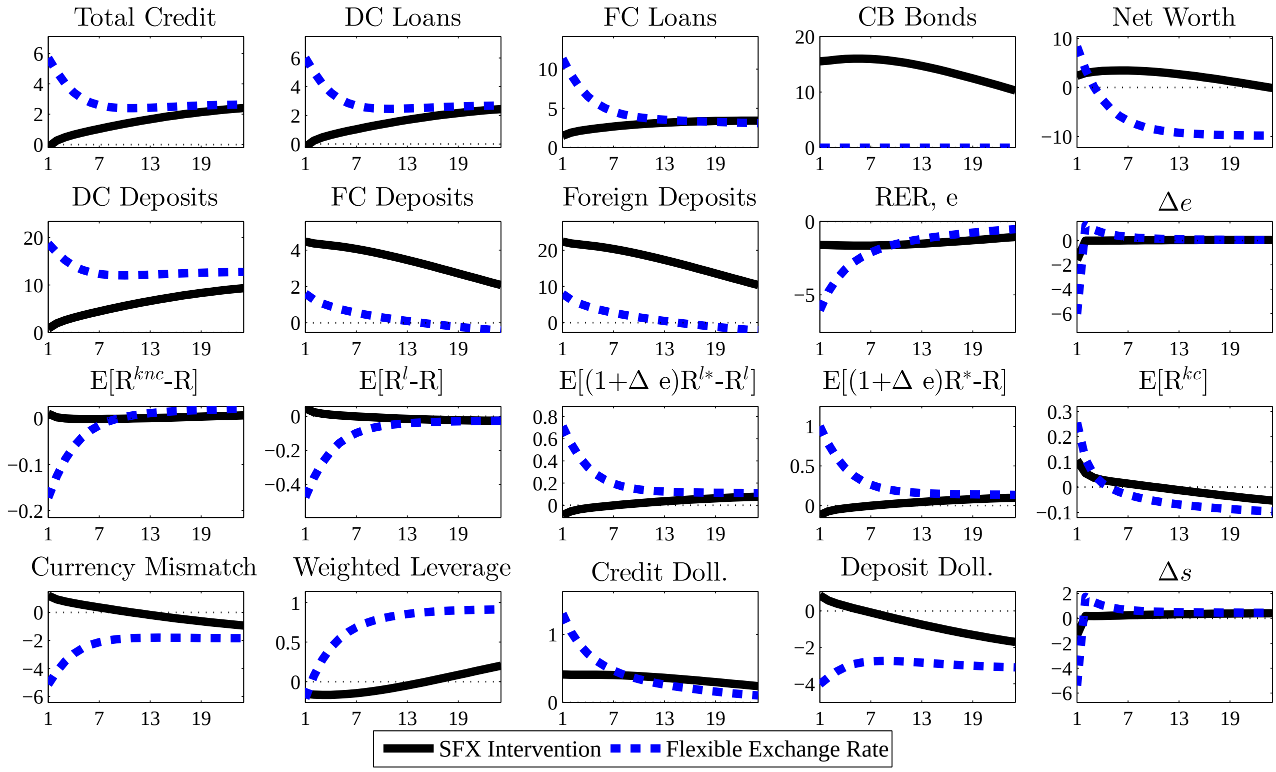
4.2 Commodity Price Shock

EMEs face volatile commodity prices that shape capital flows and domestic financial conditions. In this section, we simulate a persistent increase in commodity prices and compare the transmission mechanism of this shock under exchange rate flexibility and FX intervention. Figure 5 shows the responses of financial variables, while Figure 6 presents the response of key macroeconomic variables. The dotted line corresponds to the flexible exchange rate regime.

Under exchange rate flexibility, a persistent increase in commodity prices raises exports and a large fraction of the revenues from commodity exports remains in the economy, leading to a persistent exchange rate appreciation of around 6% at impact (see Δe in Fig. 5). The commodity sector experiences a prolonged economic boom that spreads to the rest of the economy through a significant wealth effect and a higher demand of investment goods.

The exchange rate appreciation relaxes the agency constraint that banks face via a 9% (see Fig. 5) increase in net worth, together with a significant currency mismatch fall of 5.1 percentage point right after the shock. The latter is an expansionary financial effect due to the real exchange rate appreciation. Hence, lending in both currencies rises by around 5.7%

FIGURE 5. RESPONSES OF FINANCIAL VARIABLES TO A COMMODITY PRICE SHOCK



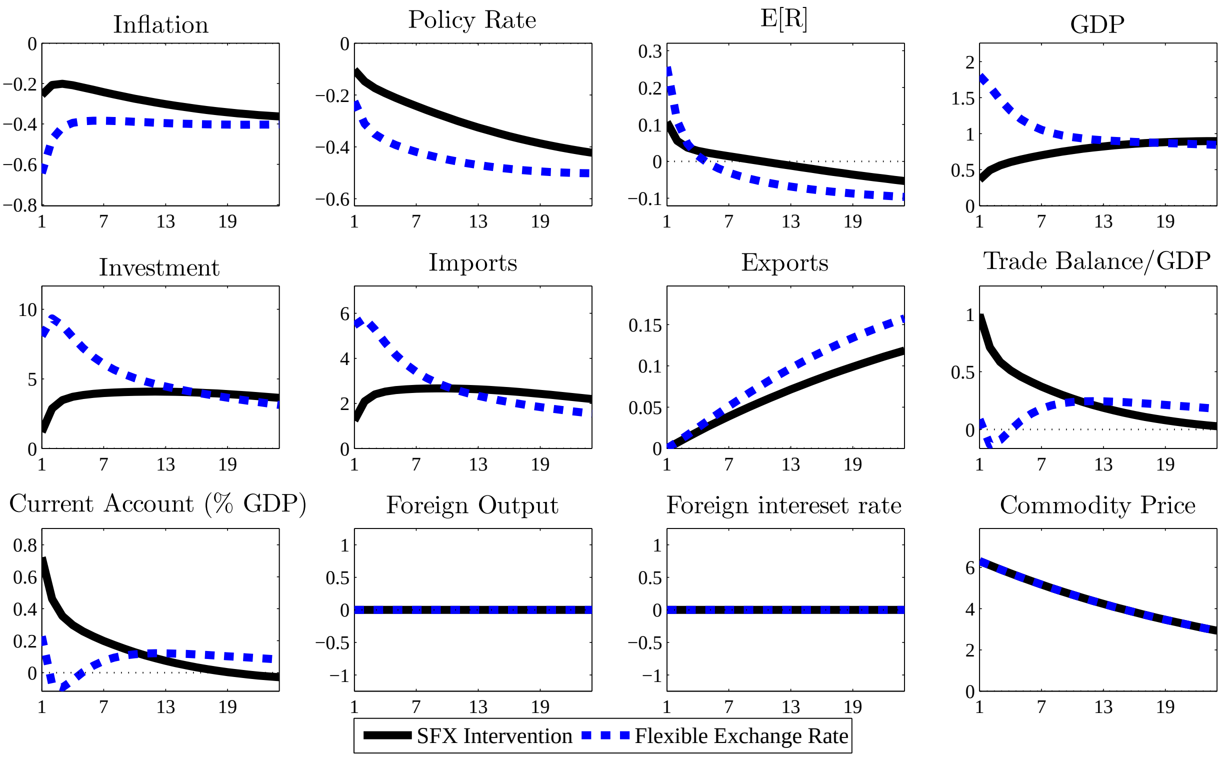
Note. The response of each quantity or index variable is presented as percent deviation from its steady-state, while the response of any rate variable is displayed in percentage-point deviations from its steady-state. e_t, Δe, and Δs denotes the real exchange rate, real depreciation, and nominal depreciation, respectively. $\mathbb{E}_t\left[\frac{e_{t+1}}{e_t}R^{l*}_{t+1} - R^l_{t+1}\right]$ and $\mathbb{E}_t\left[\frac{e_{t+1}}{e_t}R^{*}_{t+1} - R_{t+1}\right]$ measure the relative return of lending in foreign currency as well as the relative cost of borrowing in foreign currency from the point of view of banks.

at impact. Under exchange rate flexibility, agents expect a real exchange rate depreciation, implying that banks realize that borrowing in foreign currency is more expensive than in domestic currency, while lending in foreign currency is more profitable than in domestic currency. The change in the composition of banks' balance sheets is consistent with a 1.3 p.p. increase in credit dollarization and a reduction of four percentage points in deposit dollarization at impact.

The commodity boom, together with the consequent expansionary financial conditions, modify the dynamics of interest rate spreads and real macroeconomic variables. Specifically, the expected interest rate spread of domestic-currency lending relative to domestic-currency deposits falls around 0.4 percentage points (see Fig. 5), while the expected interest rate spread of foreign borrowing with respect to domestic-currency deposits raises by 1 percentage points. Investment and consumption increase persistently by around 9.3% and 0.9% at the peak of their responses, respectively (see Fig. 6). The commodity boom under a flexible exchange rate regime induces a period of persistent economic expansion, with GDP increasing in 1.8% at impact.

When FX intervention is used, the central bank accumulates FX reserves and allocates central bank riskless bonds to the banking system as a response to higher commodity prices and the appreciatory pressures on the real exchange rate. Given the binding agency problem, accumulating FX reserves significantly reduces exchange rate appreciation, thereby limiting

Figure 6. Responses of macroeconomic variables to a commodity price shock



Note. The response of each quantity or index variable is presented as percent deviation from its steady-state, while the response of any rate variable is displayed in percentage-point deviations from its steady-state.

the expansion of bank credit and the consequent expansion in macroeconomic aggregates such as consumption, investment, and GDP. As mentioned before, FX interventions operate through the exchange rate smoothing channel and the balance sheet substitution channel.

The Exchange Rate Smoothing Channel. The central bank responds to a commodity price shock by buying FX reserves, thereby modifying the net foreign asset position of the economy. As a result, exchange rate dynamics change relative to the flexible exchange rate regime. At impact, the real exchange rate appreciates by 1.6% instead of 6% (see Fig. 5). Consequently, at impact banks' net worth increases less than under free floating (2% instead of 9%). Moreover, the smoother pattern of real exchange rate modifies the costs and returns of foreign-currency borrowing and lending. When the central bank implements FX intervention, banks increase foreign borrowing together with domestic deposits, implying higher deposit dollarization relative to the flexible exchange rate regime (see the response of $\mathbb{E}_t\left[\frac{e_{t+1}}{e_t}R^{l*}_{t+1} - R^l_{t+1}\right]$ and $\mathbb{E}_t\left[\frac{e_{t+1}}{e_t}R^{*}_{t+1} - R_{t+1}\right]$ in Fig. 5). Likewise, the expected real exchange rate appreciation under FX intervention signals banks that foreign-currency lending is more profitable than lending in domestic currency. Credit dollarization increases, but less than under exchange rate flexibility.

The Balance Sheet Substitution Channel. When the central bank responds to a commodity price shock by building FX reserves, a sterilization operation is implemented simultaneously; i.e., central bank bonds are sold to maintain the domestic liquidity constant (see CB Bonds in Fig. 5). As a result, the composition and size of banks' balance sheets change, ultimately generating a crowding-out effect that limits lending resources. In particular, banks allocate their increased available funds to central bank bonds instead of lending. Banks increase their holdings of central bank bonds by 16% at the moment of the commodity shock. Accordingly, lending in both currencies increase by less than under exchange rate flexibility. The muted response of aggregate credit under the FX intervention regime is reflected in the response of interest rate spreads. Figure 5 shows that the interest rate spread of domestic-currency lending over domestic currency deposits raises around 0.1 p.p. when the central bank responds by building FX reserves instead of falling 0.5 p.p. under exchange rate flexibility.

4.3 The transmission of a permanent buildup of FX reserves

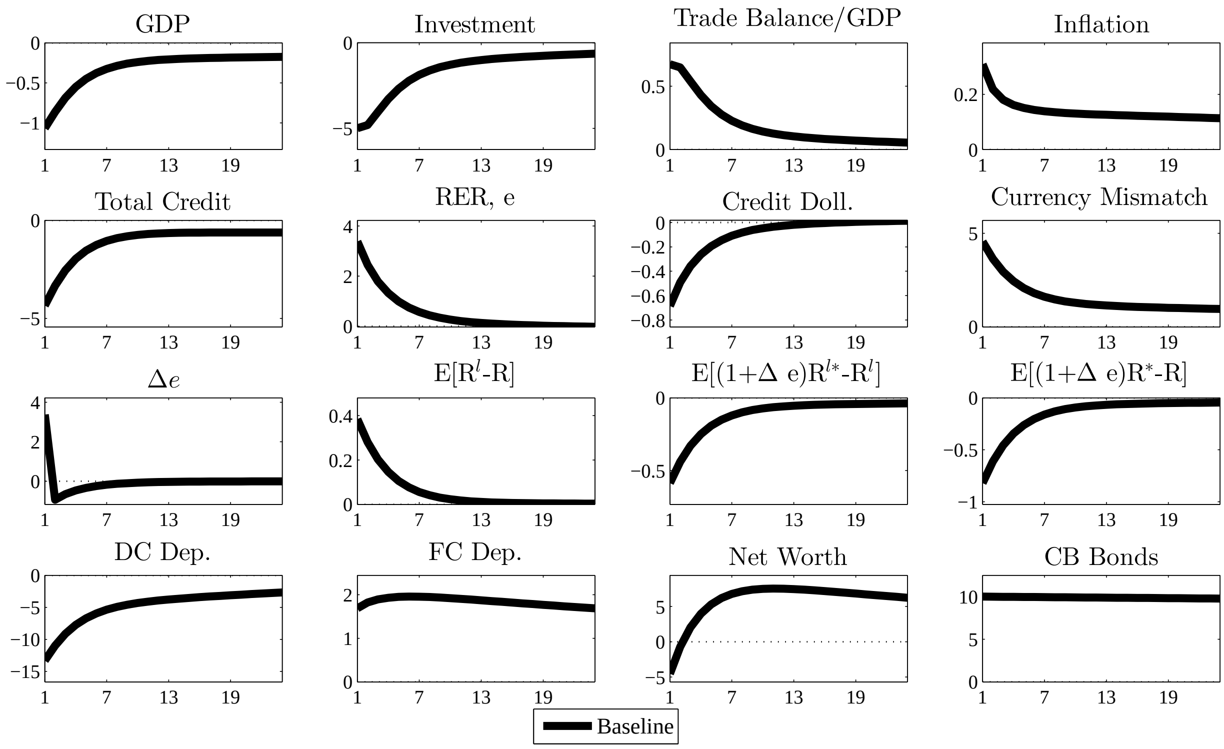
In this section, we analyze the impact of an exogenous FX intervention shock to obtain more insights about the transmission mechanism. We assume the FX intervention rule is given by the following exogenous autoregressive process:

$$\ln B_t - \ln B = \rho_B \left(\ln B_{t-1} - \ln B\right) + u_t^B, \quad \text{with } \rho_B \approx 1 \tag{55}$$

where u_t^B is interpreted as an unanticipated central bank purchase of FX reserves. Under the above process, an exogenous buildup of FX reserves has permanent effects over central bank bonds in hands of the banking system. Figure 7 shows responses to a sufficiently persistent unanticipated purchase of FX reserves together with the corresponding sterilization operation (i.e., selling of central bank bonds to the banking system). The buildup of FX reserves induces an initial real exchange rate depreciation of around 3.5% that raises inflation and the monetary policy rate as well. The trade channel triggers a corresponding trade balance surplus. The balance sheet substitution channel is such that the sterilization operation modifies the asset composition of banks' balance sheet to less lending and more central bank bonds. Finally, the purchase of FX reserves by the central bank induce a financial channel too. The real exchange rate depreciation reduces banks' net worth and raises currency mismatch at the bank level.

Consequently, domestic financial conditions worsens, which is reflected in higher interest rate spreads and lower aggregate credit. The real exchange rate dynamics is such that agents expect an appreciation right after the shock occurs. Therefore, deposit dollarization increases while credit dollarization falls. The financial and the balance sheet substitution channels outweighs the trade channel. As a result, the persistent and exogenous buildup of FX reserves push the economy to a credit crunch generating a prolonged recession.

Figure 7. Response to a Persistent Purchase of FX Reserves



Note: The response of each quantity or index variable is presented as percent deviation from its steady-state, while the response of any rate variable is displayed in percentage-point deviations from its steady-state. e_t and Δe denotes the real exchange rate and real depreciation, respectively. $\mathbb{E}_t\left[\frac{e_{t+1}}{e_t}R^{l*}_{t+1} - R^l_{t+1}\right]$ and $\mathbb{E}_t\left[\frac{e_{t+1}}{e_t}R^{*}_{t+1} - R_{t+1}\right]$ measure the relative return of lending in foreign currency as well as the relative cost of borrowing in foreign currency from the point of view of banks.

It is worth mentioning that the financial channel as well as the balance sheet substitution channel amplify the initial exogenous buildup of FX reserves shock. On the contrary, both channels work as a stabilization mechanism when FX interventions are implemented as a response to external shocks. Figure 8 summarizes the main transmission mechanisms through which FX interventions stabilize financial and macroeconomic volatility.

4.4 Generalizations of the Baseline[19]

We relax some assumptions of our baseline framework to assess whether the effectiveness of FX interventions as a response to external shocks depends on those assumptions. We compare the baseline model with the following extensions:[20]

Case 1: The steady state of the model economy is recalibrated to be consistent with a higher steady state level for the average currency mismatch of the banking system (x_t).

[19] Section To Be Updated.

[20] In Appendix B.2, we present results for an addional extension where the three assets that banks can hold enter with equal weights into the incentive compatibility constraint. Therefore, central bank bonds have a higher impact on the total amount of divertible funds and ultimately on banks' lending capacity. As a result, FX interventions are more effective in this case than in our baseline model.

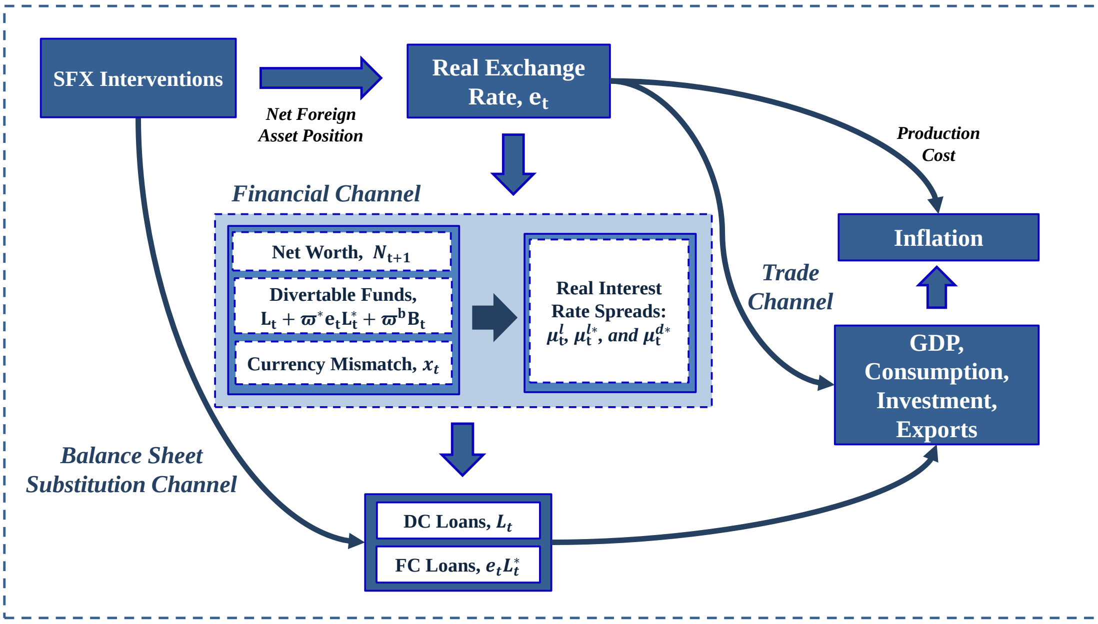
We consider an increase of five additional percentage points relative to our baseline calibration by targeting a lower foreign interest rate and a higher level of central bank bonds at the steady state.

Case 2: An economy without financial dollarization. Intermediate good producers borrow from banks only in domestic currency while households are not allowed to hold deposits with banks that are denominated in foreign currency.

Case 3: Household's demand for bank deposits in foreign currency is infinitely responsive to arbitrage opportunities.

Case 4: The size of the currency mismatch affecting bankers' ability to divert funds is assumed to be an aggregate measure of the banking system, and therefore it is taken as given at the individual level

FIGURE 8. STABILIZATION CHANNELS OF FX INTERVENTIONS



Each of these generalizations presents key features that affect the steady state of the model, as well as the effectiveness of FX interventions to mitigate the economy's response to external shocks. The steady state equilibrium for each case, including the baseline model, is presented in Table 5. The differences arise mainly from the financial block of the model (the steady state for the real sector hardly changes). We discuss the implications of each case in relation to the role of FX interventions in smoothing the economy's response to external shocks. We compute the relative accumulated responses from external shocks to key endogenous variables. More specifically, we focus on the percentage difference between relative accumulated responses of different variables under the flexible exchange regime and the FX intervention regime.[21]

The parametrization of the baseline model implies that central bank bonds are harder to deviate relative to loans (i.e., $\varpi^* > 1 > \varpi^b$). Since central bank bonds are the only sterilization instrument that the central bank is able to use, the role of FX interventions in mitigating the impact of external shocks is limited by the value of ϖ^b. However, when all the assets

[21] Additional figures and tables associated to these results are shown in appendix B.

TABLE 5. STEADY STATE EQUILIBRIUM

VARIABLE	BASELINE	Case 1 $\uparrow x_t$	Case 2 $\delta^f = D^{*,h} = 0$	Case 3 $\downarrow \kappa_{D*}$	Case 4 AGG. x_t	NOTATION
			Financial System Rates			
Capital return	8.00	7.55	8.12	8.00	8.00	$400(\mathrm{R}^{\mathrm{knc}}-1)$
DC Loan's return	6.00	5.84	6.00	6.00	6.00	$400(\mathrm{R}^{\mathrm{l}}-1)$
FC Loan's return	4.00	3.17	-	4.00	4.00	$400(\mathrm{R}^{\mathrm{l*}}-1)$
FX Bonds return	4.00	3.67	3.93	4.00	4.00	$400(\mathrm{R}^{\mathrm{b}}-1)$
Foreign Interest Rate	1.00	0.25	1.00	1.00	4.00	$400(\mathrm{R}^{*}-1)$
Deposit Interest Rate	4.00	4.00	4.00	4.00	4.00	$400(\mathrm{R}-1)$
Bank Leverage, B	1.04	1.74	1.21	1.04	1.04	ϕ^b
Bank leverage, L	3.50	3.13	6.86	3.50	3.50	ϕ^l
Bank leverage, L^*	2.59	2.32	0.00	2.59	2.59	ϕ^{l*}
Currency Mismatch	17.22	23.70	20.23	17.22	17.22	$100x$
Credit Dollarization	42.50	42.54	0.00	42.50	42.50	$100\frac{e\mathrm{L}^*}{\mathrm{L}+e\mathrm{L}^*}$
Deposit Dollarization	62.23	64.99	23.09	62.23	62.23	$100\frac{e\mathrm{D}^*}{\mathrm{D}+e\mathrm{D}^*}$
RER	1.00	1.02	1.00	1.00	1.00	e
			Sectoral Rates			
Commodity/Total Exports	60.00	60.52	59.96	60.00	60.00	$100\frac{\mathrm{Y}^{x,c}}{\mathrm{Y}^x}$
Commodity/Total Investment	16.67	16.50	16.82	16.67	16.67	$100\frac{\mathrm{I}^c}{\mathrm{I}}$
			Stock Rates			
Non Commodity Capital/GDP	2.00	2.05	1.99	2.00	2.01	$\frac{\mathrm{K}^{nc}}{4\mathrm{GDP}^{nc}}$
Commodity Capital/GDP	1.59	1.59	1.59	1.59	1.55	$\frac{\mathrm{K}^c}{4\mathrm{Y}^{x,c}}$
Stock of Capital/GDP	2.00	2.04	1.97	2.05	2.00	$\frac{\mathrm{K}}{4\mathrm{GDP}}$
Foreign Reserves/GDP	23.00	45.48	22.90	23.63	23.00	$100\frac{\mathrm{B}^{fx}}{4\mathrm{GDP}}$
			Aggregate Demand Rates			
Investment/GDP	20.00	20.42	19.70	20.55	20.00	$100\frac{\mathrm{I}}{\mathrm{GDP}}$
Public Consumption/GDP	15.00	14.83	14.94	15.41	15.00	$100\frac{\mathrm{G}}{\mathrm{GDP}}$
Consumption/GDP	58.00	57.63	58.26	56.85	58.00	$100\frac{\mathrm{C}}{\mathrm{GDP}}$
Current Account/GDP	-0.06	-0.02	0.08	-0.06	-0.24	$100\frac{\mathrm{CC}}{\mathrm{GDP}}$
Trade Balance/GDP	7.00	7.12	7.10	7.19	7.00	$100\frac{\mathrm{TB}}{\mathrm{GDP}}$

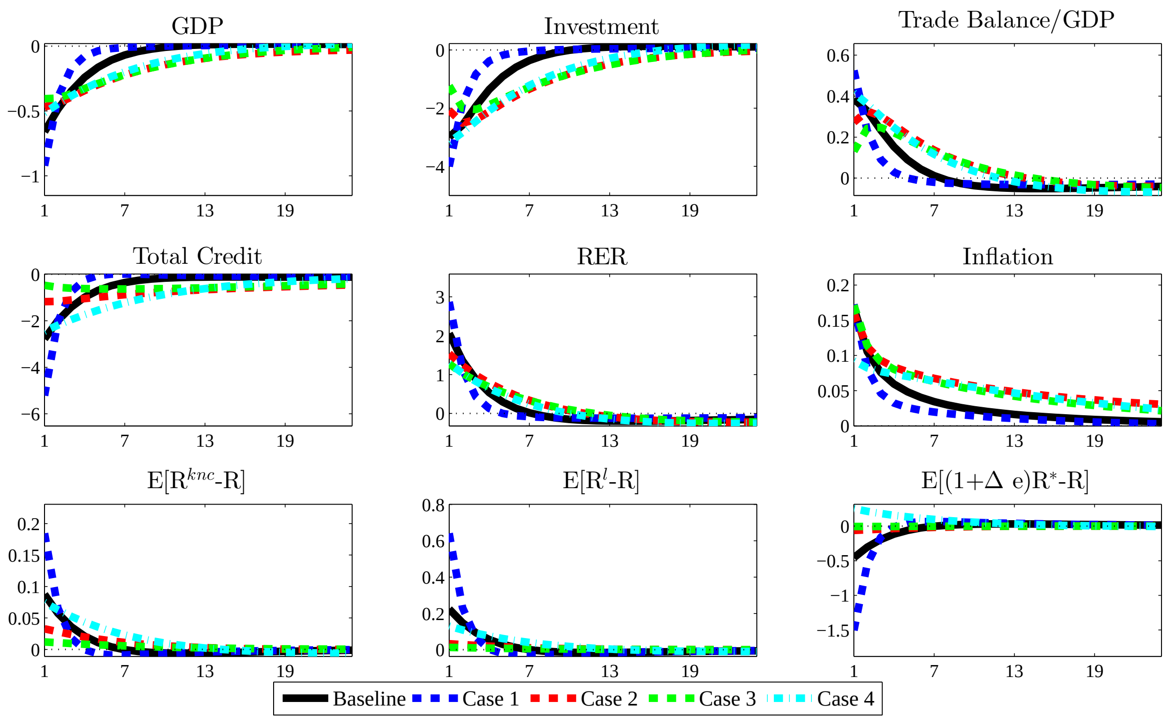
that banks can hold enter with equal weights into the incentive compatibility constraint (i.e., $\varpi^* = \varpi^b = 1$), central bank bonds have a higher impact on the total amount of divertible funds and ultimately on banks' lending capacity. As a result, FX interventions are more effective as an external shock absorber in this case than in our baseline model.

In order to measure the quantitative consequences between the model under case 1 and the baseline case, we focus on the percentage difference between the unconditional volatilities of key macroeconomic variables to an external shock under the FX intervention regime and exchange rate flexibility.

Quantitatively, our results suggest that under case 1, FX interventions relative to exchange rate flexibility stabilize the volatilities of total credit, investment, and GDP by around 86%, 72%, and 78%, respectively. In contrast, when we simulate the baseline economy, the unconditional volatilities of those variables are stabilized by 82%, 65%, and 70% accordingly (see Table 8 in Appendix B.1).

When firms do not demand foreign-currency loans (case 2), the steady-state currency mismatch size for the banking system is higher than in the baseline case (53.5% under case 2 rather than 17.2% under baseline, see Table 5). Thus, in equilibrium banks are more exposed to real exchange rate movements. In particular, banks' net worth is more sensitive to exchange rate movements in case 2. On the other hand, non-financial firms are now free of real exchange risks from financial costs and it can bound the volatility of macroeconomic variables. Which of the two effects are more relevant in our modelled economy is a quantitative matter.

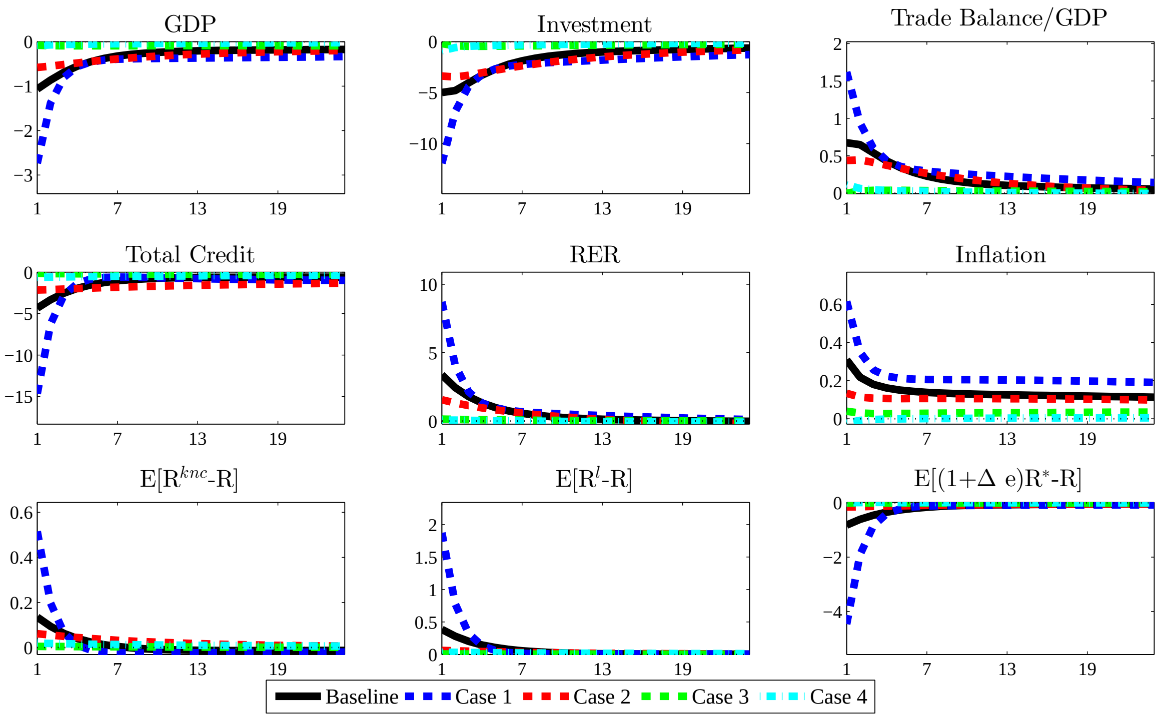
FIGURE 9. FOREIGN INTEREST RATE SHOCK UNDER FER



Note. The response of each quantity or index variable is presented as percent deviation from its steady-state, while the response of any rate variable is displayed in percentage-point deviations from its steady-state. e_t and Δe denotes the real exchange rate and real depreciation, respectively. $\mathbb{E}_t\left[\frac{e_{t+1}}{e_t}R^{l*}_{t+1} - R^l_{t+1}\right]$ and $\mathbb{E}_t\left[\frac{e_{t+1}}{e_t}R^{*}_{t+1} - R_{t+1}\right]$ measure the relative return of lending in foreign currency as well as the relative cost of borrowing in foreign currency from the point of view of banks.

In the model economy consistent with case 2, non-financial firms are only affected by real exchange rate movements through the price of imported inputs, but not through foreign-currency loans. Thus, although the banking system is more exposed to exchange rate fluctuations due to higher currency mismatch levels, firms are not financially vulnerable, since their bank debts are denominated in domestic currency. In Figure 9, we compare the impulse responses to a foreign interest rate shock between the baseline model and the model economy with no credit dollarization ($\delta^f = 0$, represented by the dotted line). In an economy with deposit dollarization but no credit dollarization, the depreciation of the real exchange rate is larger than in the case with credit and deposit dollarization (at impact the real exchange rate increases by 1.5% in case 2, compared with 1% in our baseline case). It suggests that, in case 2, FX interventions are less effective in reducing the real exchange rate depreciation when the economy faces a foreign interest rate shock. Similarly, banks' net worth, total credit, and currency mismatch level decline more in case 2 when compared with our baseline economy. As a result, the consequent recession in terms of total credit, investment, and GDP is much deeper under the case 2 which is ultimately associated with having a banking sector may be more exposed to real exchange rate fluctuations (even though the real sector are not financially exposes to exchange rate movements). In line with this finding, Table 8 in Appendix B.1 indicates that the unconditional volatility of total credit, investment, and GDP by around 69%, 52%, and 53%, instead of 82%, 65%, and 70%, respectively, suggested by our baseline parametrization.

Figure 10. FX Reserves Purchase Shock in Generalizations of the Model



Note. The response of each quantity or index variable is presented as percent deviation from its steady-state, while the response of any rate variable is displayed in percentage-point deviations from its steady-state. e_t and Δe denotes the real exchange rate and real depreciation, respectively. $\mathbb{E}_t\left[\frac{e_{t+1}}{e_t}R^{l*}_{t+1} - R^{l}_{t+1}\right]$ and $\mathbb{E}_t\left[\frac{e_{t+1}}{e_t}R^{*}_{t+1} - R_{t+1}\right]$ measure the relative return of lending in foreign currency as well as the relative cost of borrowing in foreign currency from the point of view of banks.

In case 3, banks do not internalize the effects of borrowing in foreign currency on the aggregate currency mismatch of the banking system. In this case, banks are indifferent between borrowing from domestic depositors and from abroad, implying that the standard UIP condition holds without any endogenous risk premium; i.e., $\mu_t^{d*} = 0$. Notably in this case, even though the incentive constraint binds the response of the real exchange rate to external shocks is similar under FX interventions and exchange rate flexibility. This result differs from Céspedes et al. (2017) and Chang (2019) where FX interventions are irrelevant only when the incentive compatibility constraint does not bind. Our irrelevance result is due to the indeterminacy of banks' liability composition that occurs when banks do not internalize the effect of currency mismatch over financial constraints (see Table 8 in Appendix B.1 for further quantitative results).

Additionally, we simulate an exogenous transitory sterilized purchase of FX reserves under case 3 and compare it to our baseline model. Figure 10 shows responses to this shock. It is clear that under case 3, FX interventions are irrelevant for real exchange rate dynamics even though the financial constraint binds. Moreover, in this case, the only active channel is the balance sheet substitution channel which is directly related to the sterilization leg of FX interventions. Figure 10, shows that the latter channel is not strong enough to generate significant real effects when compared to our baseline model.

5 Welfare Analysis

We conduct a policy evaluation exercise by computing the welfare gains/costs of one policy regime relative to a different regime. Each policy regime is characterized by its own time-invariant stochastic equilibrium allocation. In particular, we follow Schmitt-Grohe and Uribe (2007) and define two policy regimes denoted by $\mathcal{R}$ and $\mathcal{A}$. In particular, our benchmark regime $\mathcal{R}$ is such that the central bank has two policy instruments: monetary policy rate and FX interventions, and the degree of the endogenous response of both of them is based on our baseline parametrization. On the other hand, the alternative regime $\mathcal{A}$ assumes distinct degrees of responses for these monetary policy instruments including the absence of FX interventions, i.e., $\upsilon_e = 0$.

We define the welfare associated with the equilibrium allocation implied by our benchmark policy regime r_1 conditional on a particular state of the economy in period 0 as

$$\mathbb{W}\left(\{C_t^{\mathcal{R}}, H_t^{\mathcal{R}}\}_{t\geq 0}\right) \equiv (1-\beta)\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t \frac{1}{1-\gamma}\left(C_t^{\mathcal{R}} - \mathcal{H}C_{t-1}^{\mathcal{R}} - \frac{\zeta_0}{1+\zeta}(H_t^{\mathcal{R}})^{1+\zeta}\right)^{1-\gamma}\right]$$

where $\{C_t^{\mathcal{R}}, H_t^{\mathcal{R}}\}_{t\geq 0}$ is a contingent plan for consumption and hours under the policy regime $\mathcal{R}$. The distinct policy regimes that we consider only change the dynamics of the model economy but not its non-stochastic steady state. Therefore, we compute the welfare associated to each policy regime conditional on the initial state being the non-stochastic steady state of the model economy. The latter ensures that the economy begins from the same initial point under all possible policies. In particular, we compute the welfare gain of regime $\mathcal{A}$ relative to the benchmark policy regime $\mathcal{R}$. Let ς_{cond} denote the welfare gain/cost of adopting policy regime $\mathcal{A}$ instead of the benchmark policy regime $\mathcal{R}$ conditional that the economy is at non-stochastic steady state at time zero. The parameter ς_{cond} measures the fraction of the benchmark regime consumption process that a household would be willing to accept (or give up) to be as well off under the alternative policy regime $\mathcal{A}$ as under regime $\mathcal{R}$. Thus, ς_{cond} is implicitly defined by

$$\mathbb{W}\left(\{C_t^{\mathcal{A}}, H_t^{\mathcal{A}}\}_t\right) = \mathbb{W}\left(\{(1+\varsigma_{cond})C_t^{\mathcal{R}}, H_t^{\mathcal{R}}\}_t\right) \tag{56}$$

where $\{C_t^{\mathcal{A}}, H_t^{\mathcal{A}}\}_{t\geq 0}$ is the corresponding contingent plan for consumption and hours under the policy regime $\mathcal{A}$. Hence, if $\varsigma_{cond} > 0$ there is a welfare gain while if $\varsigma_{cond} < 0$ then there is a welfare loose under the alternative regime $\mathcal{A}$. We approximate ς_{cond} up to a second order of accuracy.

Table 6 shows the welfare gains for different combinations of monetary and FX intervention policy regimes. We change parameters ω_π and υ_e in order to study the consequences of implementing different policy rules.

Table 6. Welfare Analysis: $\varsigma_{cond} \times 100\%$

$\omega_\pi \backslash \upsilon_e$	0	2.5	5	Baseline, 9.71	20	30	50	100
1.25	−22.8	−19.1	−16.6	−12.0	−3.4	0.8	3.8	5.3
Baseline, 1.50	−6.2	−3.3	−2.0	0.0	3.4	4.8	5.6	5.9
2.00	−0.2	1.9	2.8	3.7	5.3	5.9	6.1	6.0
3.00	1.6	3.4	4.1	4.7	5.7	6.1	6.2	6.0
5.00	2.1	3.8	4.4	4.9	5.9	6.2	6.2	6.0

Note. The parameter ω_π controls the policy rate response to fluctuations in inflation. Parameter υ_e measures the response of FX interventions to real exchange rate deviations. For each combination of (ω_π, υ_e) we compute ς_{cond} which is defined above. Only external shocks are considered.

Tab. 6 shows the welfare gains from different combinations of monetary policy rate and FX intervention rules. We want to emphasize two relevant remarks from this table. First, conditional to external shocks, FX interventions and interest rate policies are effective in reducing macroeconomic volatility and increasing social welfare in a big region of the parameter space associated to both policy regimes (i.e., υ_e and ω_π). Second, our model suggests that, given the Taylor coefficient at its baseline level, not responding to the real exchange rate by implementing FX interventions ($\upsilon_e = 0$) would cause a welfare loose of 6.2% in consumption. These remarks justify the actively use of FX interventions as a additional monetary policy tool aimed to smooth real exchange rate dynamics.

In a similar fashion, we compute welfare gains for each of the generalizations studied in Section xx. For case 1, the welfare loss under the free-floating exchange rate regime, relative to the FX intervention one, is higher than in the baseline. This result is consistent with the higher FX effectiveness in case 1 as discussed in the previous section. In particular, keeping the degree of the monetary policy rate response to inflation variations ($\omega_\pi = 1.50$), the associated welfare loss when the central bank does not implement FX intervention is 17.2% in terms of consumption rather than 6.2% under the baseline case (see Tab. 10 in appendix B).

Likewise, Tab. 11 in appendix B presents welfare gains associated with case 2 for different combinations of parameters in the Taylor rule as well as in the FX intervention rule. Consistent with the discussion in Section xx, the welfare loss associated with the exchange rate flexibility is lower than the corresponding welfare loss in the baseline model (2.4% in case 2 compared to 6.2% of consumption in the baseline model). Finally, for the case 3 and 4, where FX interventions is barely effective because household's arbitrage power and/or UIP holds, welfare losses in terms of consumption under the free-floating regime is negligible compared in comparison of our baseline economy (0.1-0.2% instead of 6.2%, see Tab. 12 and Tab. 13 in appendix B).

6 Concluding Remarks

In this paper we have proposed a macroeconomic model with financial frictions for a small open economy to analyze and quantify the effectiveness of FX interventions in stabilizing the impact of external shocks. FX interventions are modeled as an unconventional monetary policy tool that operates simultaneously with the conventional policy rate tool. More specifically, in our model FX interventions are considered a balance sheet policy induced by an agency problem between banks and their investors (i.e, domestic depositors and foreign lenders). Three key assumptions are important for our results. First, the severity of banks' agency problem depends directly on a measure of currency mismatch at the bank level. Second, the banking system is partially dollarized on both sides of its balance sheet and exposed to potential currency mismatches. On one hand, intermediate good producers must borrow a bundle of loan services from banks in order to produce. The composition of this bundle consists of a combination between domestic and foreign currency denominated loans. On the other hand, households are allowed to hold deposits with banks that are denominated in domestic and foreign currency. But we introduce limits on household foreign currency denominated deposits as a way to capture incomplete arbitrage. Third, FX intervention is such that the central bank leans against the wind with respect to exchange rate fluctuations but in a sterilized manner.

Our results shed light on the transmission mechanism of FX interventions. In particular, we highlight two reinforcing effects when responding to external shocks: the exchange rate smoothing channel and the balance sheet substitution channel or crowding out effect over bank lending. The former channel is active whenever banks and households are not able to seize arbitrage opportunities between domestic and foreign currency denominated deposits and assets implying endogenous deviations from UIP. Instead, if either banks or households are able to engage in frictionless arbitrage between domestic and foreign currency denominated asset returns, the standard UIP equation holds and this channel is no longer active. On the other hand, the balance sheet substitution channel stems from the sterilization operation associated to FX interventions which modifies the supply of central bank bonds in banks' balance sheet and, with it, their asset composition. Our quantitative results suggest that the latter channel is less significant than the former one.

An intereseting result arises when banks do not internalize the effects of borrowing in foreign currency on the aggregate currency mismatch of the banking system. In this case, banks are indifferent between borrowing from domestic depositors and from abroad, implying that the standard UIP condition holds without any endogenous risk premium. As a result, FX interventions are less effective in stabilizing the economy in the presence of external shocks. Notably in this case, even though the incentive constraint binds the response of the real exchange rate to external shocks is the same under FX interventions and exchange rate flexibility. This result differs from Céspedes et al. (2017) and Chang (2019) where FX interventions are irrelevant only when the incentive compatibility constraint does not bind. In sum, in our framwork, for FX interventions to affect significantly the real exchange rate and excess returns along with the aggregate equilibrium of the economy, limits to arbitrage between domestic and foreign currency denominated assets and liabilities must be present for both, households and banks.

We consider that the financial friction view of FX interventions needs further research. For instance, it differs from the unconventional monetary policy framework for closed economies in several ways. First, FX interventions have been implemented effectively even in normal times in EMEs, contrary to the unconventional monetary policy tools studied in the context of closed economies. In the latter case, once the effective lower bound is reached, unconventional tools may be deployed. Second, what really matters for EMEs is how tight financial constraints are, and not necessarily if they bind or not. Third, in practice, the communication of FX interventions is at odds with the communication of unconventional policies in closed economies. For example, it seems that there is much less forward guidance associated with FX interventions than with QE or LSAP. Finally, the effective lower bound for EMEs may not only be related to the nominal interest rate, but also to a non-negative amount of official FX reserves needed to implement FX interventions within an inflation-targeting regime.

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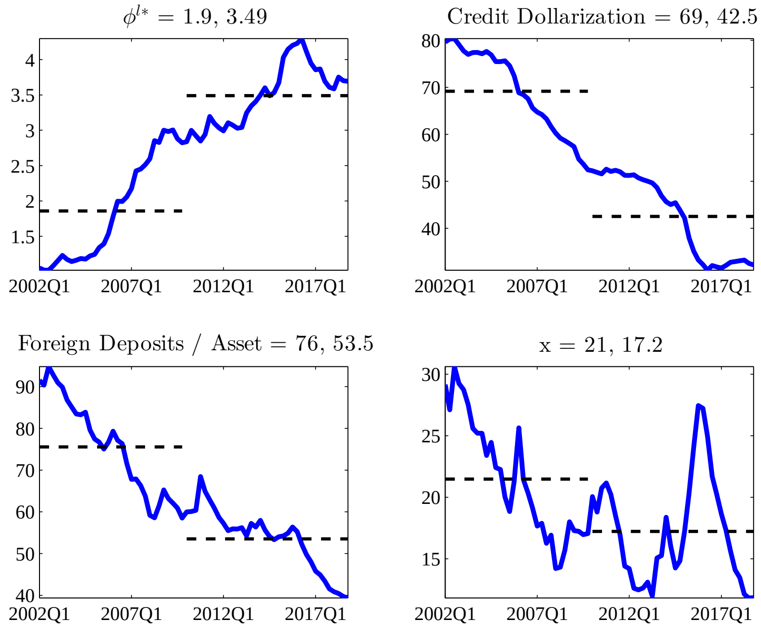
A Parametrization

We set the steady-state targets based on Peruvian banking system data. First, calibrate the consolidated balance sheet of the banking system in the model using data for Peru to obtain historical averages for the aggregate currency mismatch level and foreign currency liabilities as a fraction of total assets. We use data on domestic currency credit for L_t, dollar denominated credit for L_t^* and total banking investment for B_t. We use data on banks' net worth for N_t and the sum of foreign currency deposits and external liabilities for measuring D_t^*. Figure 11 plots the evolution of the bank's balance sheet composition that we used to fix the model's steady-state variables.

Figure 11. Bank's Balance Sheet Composition



Note. We use data on domestic currency credit for L_t, dollar denominated credit for L_t^* and total banking investment for B_t. We use data on banks' net worth for N_t and the sum of foreign currency deposits and external liabilities for measuring D_t^*.

Moreover, we use the average of domestic (foreign) currency prime, corporate, and big company loan's interest rate as our measure of domestic (foreign) currency lending return. Figure 12 shows the Peruvian banking system interest rate spread. Similarly, Figure 13 presents the aggregate real ratios used to fix the demand side steady state of the economy.

Finally, Table 7 summarizes the baseline parametrization used to fix some steady state targets.

FIGURE 12. BANKING SYSTEM INTEREST RATE SPREAD

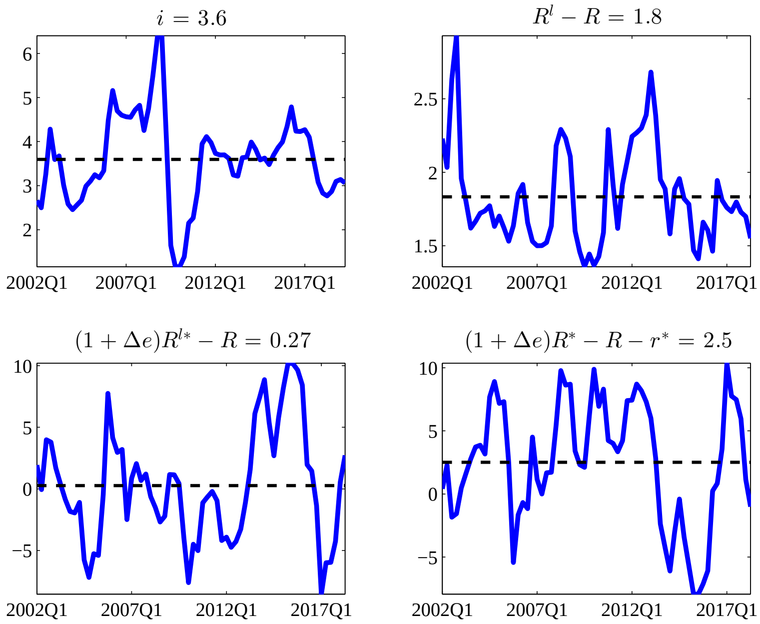


FIGURE 13. REAL AGGREGATE RATIOS

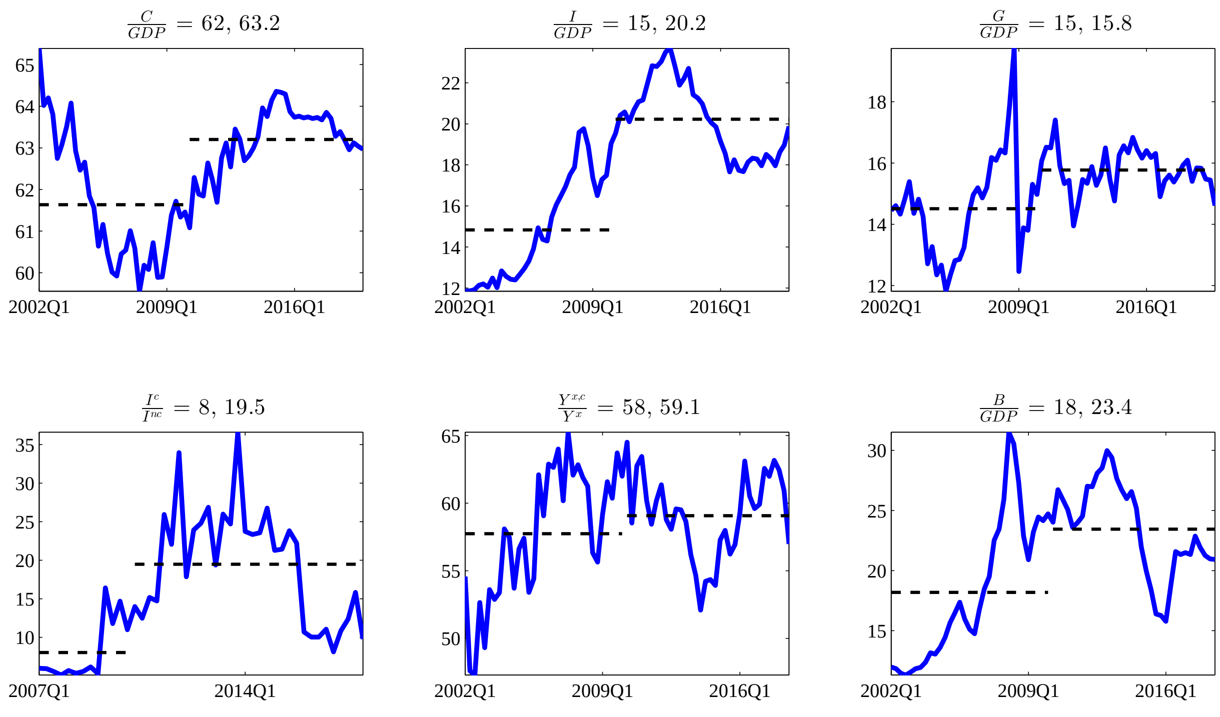


Table 7. Targeted Parametrization

Targets			Parameters		
Description	**Variable**	**Value**	**Description**	**Parameter**	**Value**
Banking Sector					
Foreign Interest Rate	R^*	$1.01^{1/4}$	Foreign Rate	R^*	1.01
Domestic Interest Rate	R	$1.04^{1/4}$	Subjective Discount Factor	β	0.99
Domestic C. Loans Return	R^l	$1.06^{1/4}$	Foreign C. Loans Participation	ϖ^*	1.42
Foreign C. Loans Return	R^{l*}	$1.04^{1/4}$	Bonds Loans Participation	ϖ^b	0.21
CB Bond Return	R^b	$1.04^{1/4}$	Banker's Survival Rate	σ	0.95
Foreign Liability to Asset	$\frac{eD^*}{A}$	53.5%	Moral Hazard 1	θ	0.68
Domestic C. Leverage	$\frac{L}{N}$	3.50	Moral Hazard 2	$\varkappa$	7.54
Credit Dollarization	$\frac{eL^*}{L+eL^*}$	42.5%	DC Bias in Loans	δ^f	0.42
Non-Banking Sector					
Capital Return	R^k	$1.08^{1/4}$	Loans Aggregator Scale	A^e	1.96
Worked Hours	H	1/3	Labor Disutility Scale	ζ_0	0.87
Real Exchange Rate	e	1	Foreign Output	Y^*	0.14
Commodity Price	p^{wc}	1	Commodity Price	p^{wc}	1.00
GDP	GDP	1	Non Commodity Productivity	A^{nc}	0.45
Commodity to Total Export	$\frac{Y^{x,c}}{Y^x}$	60%	Commodity Returns to Scale	α^c	0.22
Commodity to Non Commodity Inv.	$\frac{I^c}{I^{nc}}$	20%	Commodity Sector Productivity	A^c	0.20
Share of Capital financed by households	$\frac{\mathcal{S}}{K^{nc}}$	20%	Capital Share	α^k	0.32
Share of Foreign FC Deposits	$\frac{D^{f,*}}{D^+}$	20%	Ef. Household's Capital	$\overline{\mathcal{S}}$	-0.08
Consumption to GDP	$\frac{C}{GDP}$	58%	Ef. Household's FC Deposits	$\overline{D}^{h,*}$	2.69
Investment to GDP	$\frac{I}{GDP}$	20%	Imported Input Share	α^m	0.30
Gov. Purchases to GDP	$\frac{G}{GDP}$	15%	Gov. Expenditure	G	0.15
FX Reserves to GDP - Anual	$\frac{eF}{4\times GDP}$	23%	CB Bonds	B	0.92

B Additional Tables and Figures

B.1 Additional Tables

Table 8. Unconditional Volatilities

	Baseline			Case 1			Case 2			Case 3			Case 4		
	FXI (a)	FER (b)	(a)/(b)	FXI (a)	FER (b)	(a)/(b)	FXI (a)	FER (b)	(a)/(b)	FXI (a)	FER (b)	(a)/(b)	FXI (a)	FER (b)	(a)/(b)
RER	2.36	7.32	0.32	1.06	8.89	0.12	1.90	3.99	0.48	1.66	1.70	0.98	1.38	1.41	0.98
Real Depreciation	1.79	6.76	0.26	0.77	9.89	0.08	1.52	3.29	0.46	1.36	1.40	0.97	1.18	1.21	0.98
GDP	0.68	2.24	0.30	0.32	2.62	0.12	0.53	1.34	0.40	0.49	0.60	0.83	0.37	0.52	0.71
Investment	4.21	11.93	0.35	1.84	11.90	0.15	3.29	8.03	0.41	2.60	2.98	0.87	3.41	4.10	0.83
Consumption	0.23	0.25	0.93	0.18	0.24	0.76	0.20	0.23	0.87	0.11	0.12	0.92	0.08	0.08	0.99
Total Credit	1.25	6.79	0.18	0.69	11.45	0.06	1.61	3.00	0.54	0.42	0.67	0.64	3.76	4.44	0.85
Inflation	0.29	0.70	0.42	0.14	0.76	0.18	0.25	0.38	0.64	0.18	0.20	0.90	0.25	0.24	1.08
Currency Mismatch	2.11	6.07	0.35	1.66	7.99	0.21	2.33	0.89	2.61	0.33	0.25	1.33	2.68	2.60	1.03

Note. ”FXI” is the abbreviation for *Foreign Exchange Intervention* regimen and ”FER” for the *Flexible Exchange Rate* regimen. The computation only considers external shock volatilities and is based on 1500 replications of 120 periods simulated trajectories.

Table 9. Welfare Analysis: $\varsigma_{cond} \times 100\%$ - Perfect Substitutes

$\omega_\pi \backslash \upsilon_e$	0	2.5	5	Baseline, 9.71	20	30	50	100
1.25	−24.5	−19.5	−16.1	−9.4	−1.4	1.5	3.5	4.6
Baseline, 1.50	−8.9	−4.6	−2.9	0.0	3.2	4.2	4.7	5.0
2.00	−3.2	0.2	1.2	2.8	4.4	4.9	5.1	5.1
3.00	−1.4	1.6	2.3	3.5	4.7	5.0	5.1	5.2
5.00	−0.8	1.9	2.6	3.7	4.8	5.1	5.1	5.2

Note. The parameter ω_π controls the policy rate response to fluctuations in inflation. Parameter υ_e measures the response of FX interventions to real exchange rate deviations. For each combination of (ω_π, υ_e) we compute ς_{cond} which is defined above. Only external shocks are considered.

Table 10. Welfare Analysis: $\varsigma_{cond} \times 100\%$ - Case 1

$\omega_\pi \backslash \upsilon_e$	0	2.5	5	Baseline, 9.71	20	30	50	100
1.25	−33.8	−23.1	−16.9	−7.5	0.4	2.5	3.8	4.5
Baseline, 1.50	−17.2	−7.1	−4.0	0.0	3.1	3.8	4.2	4.6
2.00	−10.3	−1.7	−0.1	2.1	3.8	4.1	4.3	4.6
3.00	−7.7	−0.2	1.0	2.7	3.9	4.2	4.4	4.6
5.00	−6.6	0.2	1.3	2.8	4.0	4.2	4.4	4.6

Note. The parameter ω_π controls the policy rate response to fluctuations in inflation. Parameter υ_e measures the response of FX interventions to real exchange rate deviations. For each combination of (ω_π, υ_e) we compute ς_{cond} which is defined above. Only external shocks are considered.

Table 11. Welfare Analysis: $\varsigma_{cond} \times 100\%$ - Case 2

$\omega_\pi \backslash \upsilon_e$	0	2.5	5	Baseline, 9.71	20	30	50	100
1.25	−14.5	−13.1	−11.8	−9.1	−4.0	−0.9	1.7	3.1
Baseline, 1.50	−2.4	−1.7	−1.1	0.0	1.9	2.8	3.4	3.6
2.00	1.2	1.7	2.1	2.7	3.5	3.8	3.9	3.7
3.00	2.2	2.6	2.9	3.4	3.9	4.1	4.0	3.8
5.00	2.5	2.9	3.2	3.5	4.0	4.1	4.0	3.8

Note. The parameter ω_π controls the policy rate response to fluctuations in inflation. Parameter υ_e measures the response of FX interventions to real exchange rate deviations. For each combination of (ω_π, υ_e) we compute ς_{cond} which is defined above. Only external shocks are considered.

Table 12. Welfare Analysis: $\varsigma_{cond} \times 100\%$ - Case 3

$\omega_\pi \backslash \upsilon_e$	0	2.5	5	Baseline, 9.71	20	30	50	100
1.25	−4.5	−4.2	−4.0	−3.6	−3.0	−2.7	−2.7	−3.7
Baseline, 1.50	−0.2	−0.2	−0.1	0.0	−0.0	−0.3	−1.1	−3.3
2.00	0.9	0.9	0.9	0.9	0.8	0.4	−0.7	−3.1
3.00	1.2	1.2	1.2	1.2	1.0	0.6	−0.5	−3.1
5.00	1.2	1.3	1.3	1.3	1.0	0.6	−0.5	−3.0

Note. The parameter ω_π controls the policy rate response to fluctuations in inflation. Parameter υ_e measures the response of FX interventions to real exchange rate deviations. For each combination of (ω_π, υ_e) we compute ς_{cond} which is defined above. Only external shocks are considered.

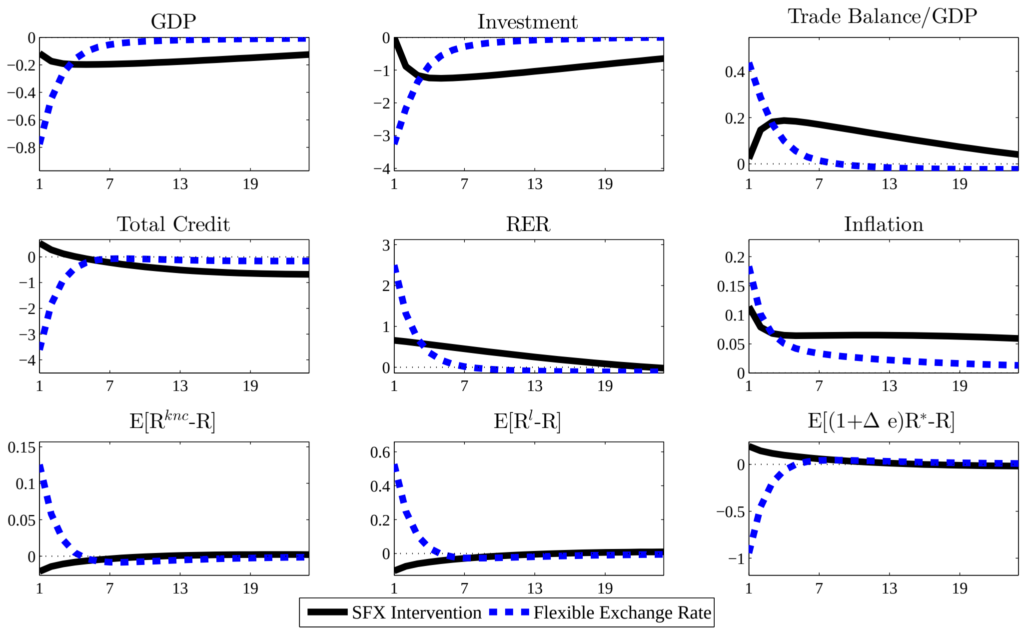
TABLE 13. WELFARE ANALYSIS: $\varsigma_{cond} \times 100\%$ - CASE 4

$\omega_\pi \backslash \upsilon_e$	0	2.5	5	Baseline, 9.71	20	30	50	100
1.25	−3.9	−3.8	−3.8	−3.8	−3.7	−3.7	−3.5	−2.2
Baseline, 1.50	−0.1	−0.0	−0.0	0.0	−0.0	−0.1	−0.2	−0.2
2.00	0.9	0.9	1.0	1.0	0.9	0.9	0.8	0.5
3.00	1.2	1.2	1.2	1.2	1.2	1.2	1.1	0.8
5.00	1.3	1.3	1.3	1.3	1.3	1.2	1.2	0.9

Note. The parameter ω_π controls the policy rate response to fluctuations in inflation. Parameter υ_e measures the response of FX interventions to real exchange rate deviations. For each combination of (ω_π, υ_e) we compute ς_{cond} which is defined above. Only external shocks are considered.

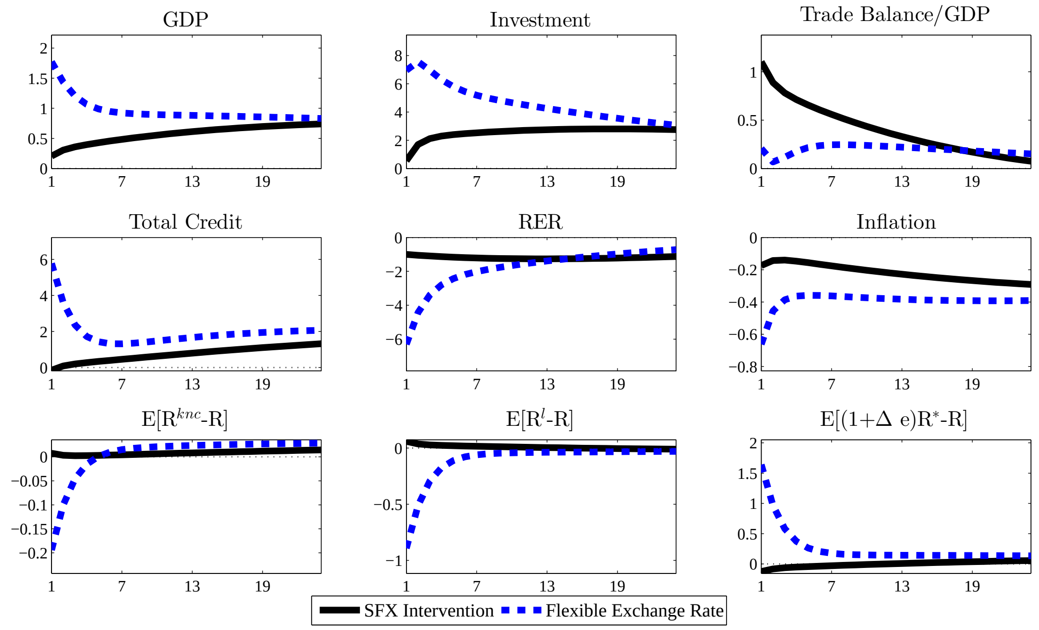
B.2 FIGURES FOR CASE 0: PERFECT SUBSTITUION AMONG BANK'S ASSET

FIGURE 14. FOREIGN INTEREST RATE SHOCK: PERFECT SUBSITUTION IN BANK'S ASSETS



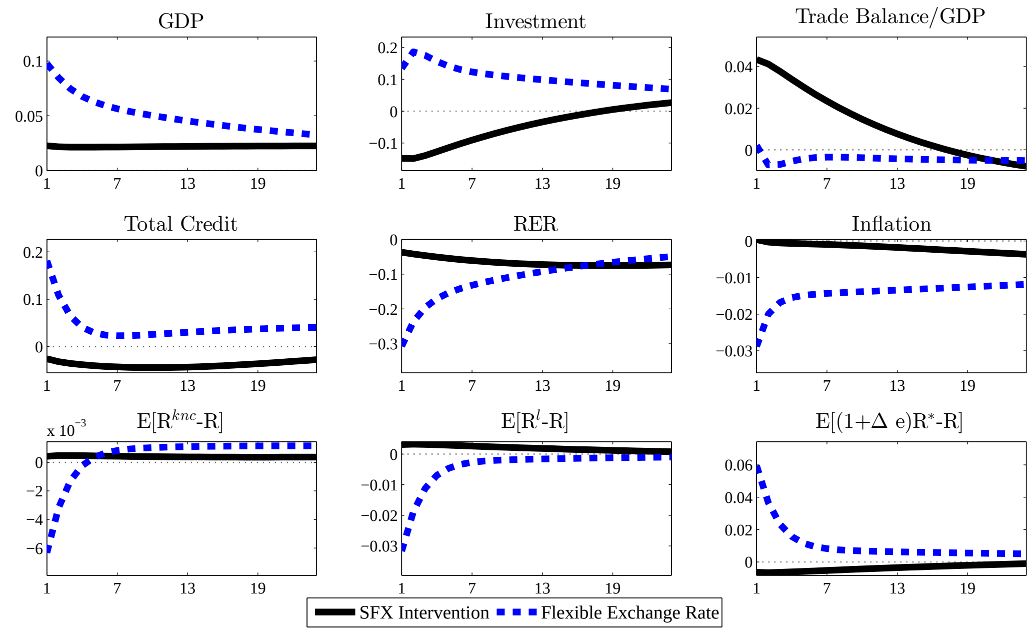
Note. The response of each quantity or index variable is presented as percent deviation from its steady-state, while the response of any rate variable is displayed in percentage-point deviations from its steady-state. $\mathbb{E}_t\left[\frac{e_{t+1}}{e_t}R^*_{t+1} - R_{t+1}\right]$ measures the relative cost of borrowing in foreign currency from the point of view of banks.

FIGURE 15. COMMODITY PRICE SHOCK: PERFECT SUBSITUTION IN BANK'S ASSETS



Note. The response of each quantity or index variable is presented as percent deviation from its steady-state, while the response of any rate variable is displayed in percentage-point deviations from its steady-state. $\mathbb{E}_t\left[\frac{e_{t+1}}{e_t}R^*_{t+1} - R_{t+1}\right]$ measures the relative cost of borrowing in foreign currency from the point of view of banks.

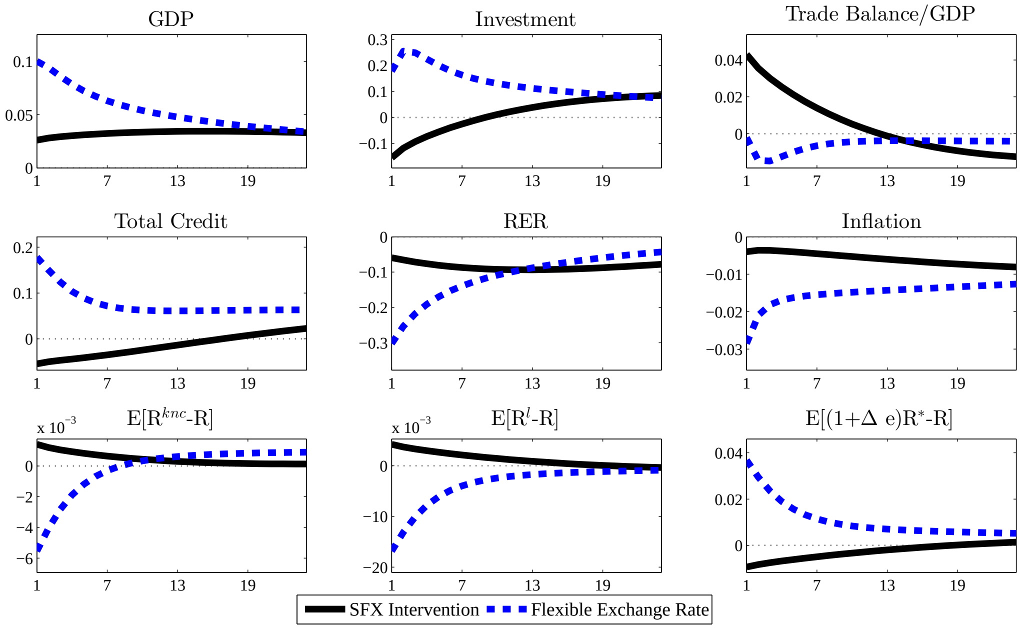
FIGURE 16. GLOBAL DEMAND SHOCK: PERFECT SUBSITUTION IN BANK'S ASSETS



Note. The response of each quantity or index variable is presented as percent deviation from its steady-state, while the response of any rate variable is displayed in percentage-point deviations from its steady-state. $\mathbb{E}_t\left[\frac{e_{t+1}}{e_t}R^*_{t+1} - R_{t+1}\right]$ measures the relative cost of borrowing in foreign currency from the point of view of banks.

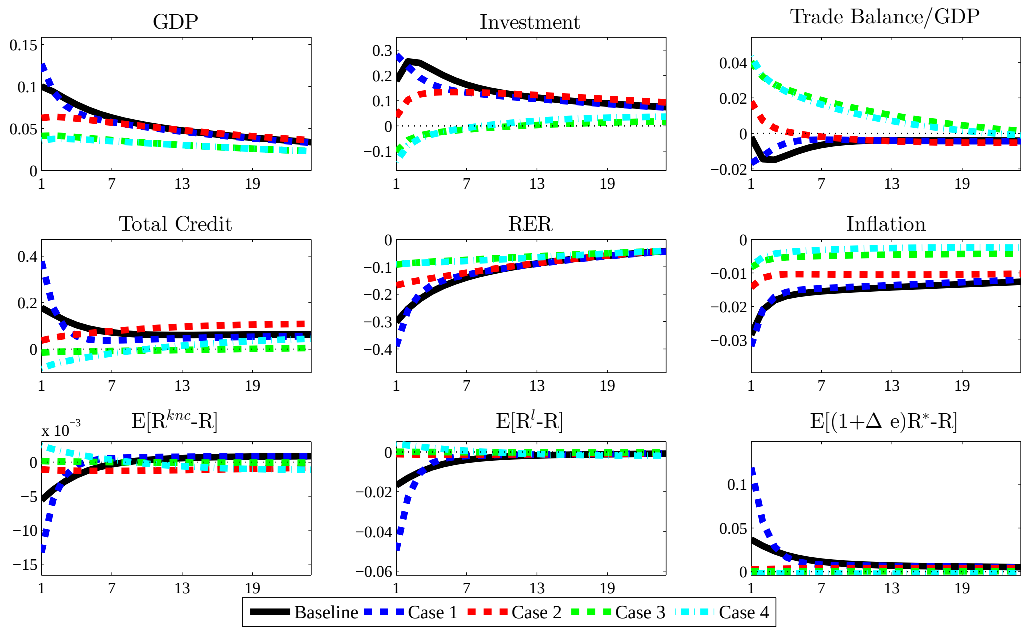
B.3 Figures for Foreign Demand Shock

Figure 17. Baseline



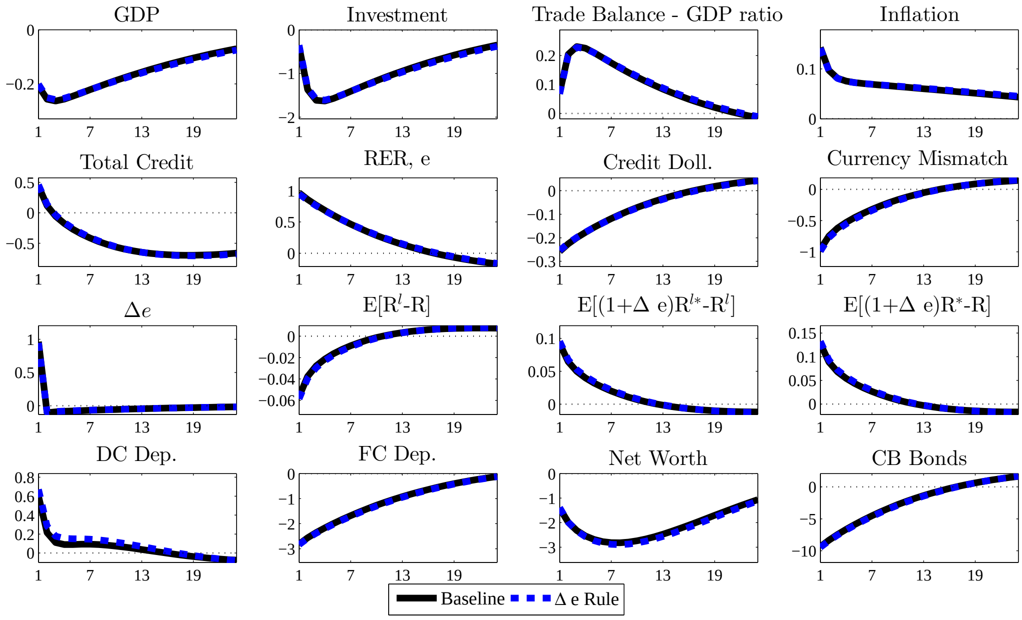
Note. The response of each quantity or index variable is presented as percent deviation from its steady-state, while the response of any rate variable is displayed in percentage-point deviations from its steady-state. $\mathbb{E}_t\left[\frac{e_{t+1}}{e_t}R^*_{t+1}-R_{t+1}\right]$ measures the relative cost of borrowing in foreign currency from the point of view of banks.

Figure 18. Generalizations of the Model under FER



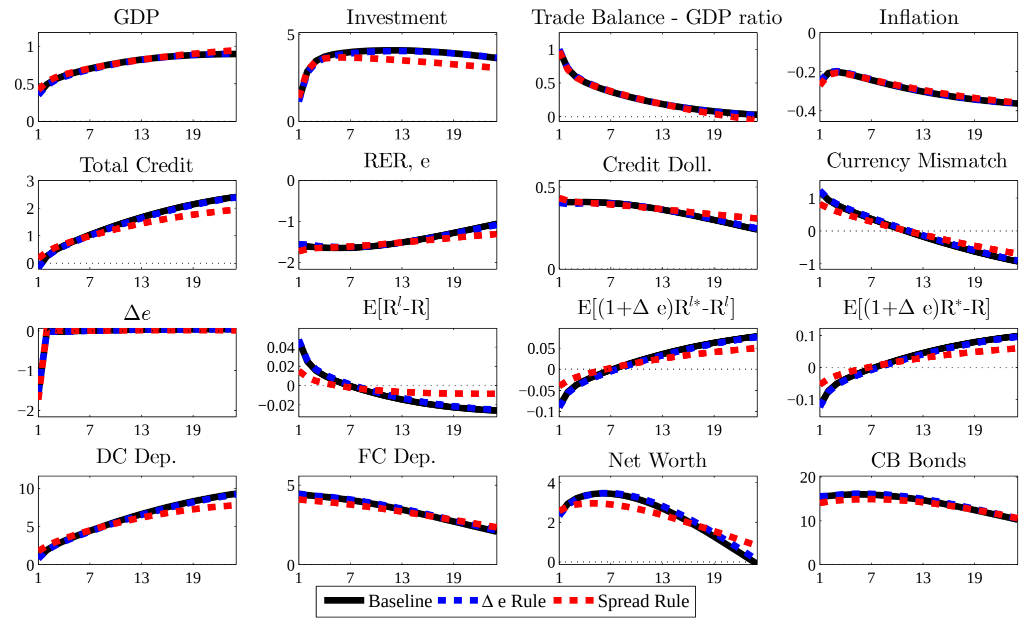
B.4 Distinct FX Intervention Rules

Figure 19. Foreign Interest Rate Shock



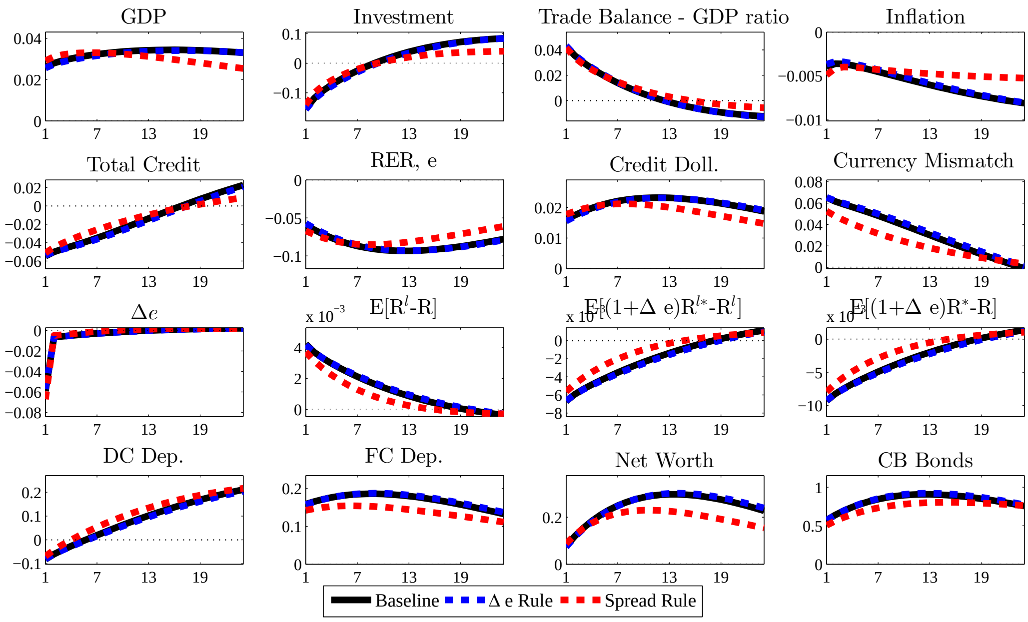
Note. "Δe Rule" considers $\rho_B = 0.999$ and $\upsilon_{\Delta e} = 10$. The response of each quantity or index variable is presented as percent deviation from its steady-state, while the response of any rate variable is displayed in percentage-point deviations from its steady-state. $\mathbb{E}_t\left[\frac{e_{t+1}}{e_t}R^*_{t+1} - R_{t+1}\right]$ measures the relative cost of borrowing in foreign currency from the point of view of banks.

Figure 20. Commodity Price Shock



Note. "Δe Rule" considers $\rho_B = 0.999$ and $\upsilon_{\Delta e} = 10$ and "Spread Rule" calibrates $\rho_B = 0.999$ and $\upsilon_{\text{spread}} = 7$. The response of each quantity or index variable is presented as percent deviation from its steady-state, while the response of any rate variable is displayed in percentage-point deviations from its steady-state. $\mathbb{E}_t\left[\frac{e_{t+1}}{e_t}R^*_{t+1} - R_{t+1}\right]$ measures the relative cost of borrowing in foreign currency from the point of view of banks.

Figure 21. Foreign Output Shock



Note. "Δe Rule" considers $\rho_B = 0.999$ and $\upsilon_{\Delta e} = 10$ and "Spread Rule" calibrates $\rho_B = 0.999$ and $\upsilon_{\text{spread}} = 7$. The response of each quantity or index variable is presented as percent deviation from its steady-state, while the response of any rate variable is displayed in percentage-point deviations from its steady-state. $\mathbb{E}_t\left[\frac{e_{t+1}}{e_t}R^*_{t+1} - R_{t+1}\right]$ measures the relative cost of borrowing in foreign currency from the point of view of banks.

C Model Solution

C.1 The Financial System

Solving Bank's Problem. Recursive version for banker's problem:

$$V_t = \max_{l_t, l_t^*, b_t, d_t, d_t^*} \mathbb{E}_t\left[\Lambda_{t,t+1}\left\{(1-\sigma)n_{t+1} + \sigma V_{t+1}\right\}\right]$$

subject to:

$$l_t + e_t l_t^* + b_t = n_t + d_t + e_t d_t^*$$

$$n_{t+1} = R^l_{t+1} l_t + R^{l*}_{t+1} e_{t+1} l_t^* + R^b_{t+1} b_t - R_{t+1} d_t - e_{t+1} R^*_{t+1} d_t^*$$

$$x_t = \frac{e_t d_t^* - e_t l_t^*}{l_t + e_t l_t^* + b_t}$$

$$V_t \geq \Theta(x_t)\left[l_t + \varpi^* e_t l_t^* + \varpi^b b_t\right]$$

Let $\psi_t = \frac{V_t}{n_t}$, $\phi^l_t = \frac{l_t}{n_t}$, $\phi^l_t = \frac{e_t l_t^*}{n_t}$, and $\phi^b_t = \frac{b_t}{n_t}$, then the objective function can be rewritten as

$$\psi_t = \mathbb{E}_t\left[\Lambda_{t,t+1}(1 - \sigma + \sigma\psi_{t+1})\frac{n_{t+1}}{n_t}\right]$$

Using the law of motion for bank's net worth, we can rearrange:

$$\frac{n_{t+1}}{n_t} = R^l_{t+1}\frac{l_t}{n_t} + R^{l*}_{t+1}\frac{e_{t+1}}{e_t}\frac{e_t l_t^*}{n_t} + R^b_{t+1}\frac{b_t}{n_t} - R_{t+1}\frac{d_t}{n_t}$$

$$
\begin{aligned}
&- R^*_{t+1}\frac{e_{t+1}}{e_t}\frac{(l_t+e_tl^*_t+b_t)}{n_t}\frac{(e_td^*_t-e_tl^*_t+e_tl^*_t)}{l_t+e_tl^*_t+b_t}\\
\frac{n_{t+1}}{n_t} &= R^l_{t+1}\phi^l_t+\frac{e_{t+1}}{e_t}(R^{l*}_{t+1}-R^*_{t+1})\phi^{l*}_t+R^b_{t+1}\phi^b_t-R_{t+1}\left[\frac{l_t}{n_t}+\frac{e_tl^*_t}{n_t}\right.\\
&\left.+\frac{b_t}{n_t}-1-\frac{e_td^*_t}{n_t}\right]-\frac{e_{t+1}}{e_t}R^*_{t+1}\left[\phi^l_t+\phi^{l*}_t+\phi^b_t\right]x_t\\
\frac{n_{t+1}}{n_t} &= R^l_{t+1}\phi^l_t+\frac{e_{t+1}}{e_t}(R^{l*}_{t+1}-R^*_{t+1})\phi^{l*}_t+R^b_{t+1}\phi^b_t\\
&-R_{t+1}\left[\phi^l_t+\phi^b_t-1-\left[\phi^l_t+\phi^{l*}_t+\phi^b_t\right]x_t\right]-\frac{e_{t+1}}{e_t}R^*_{t+1}\left[\phi^l_t+\phi^{l*}_t+\phi^b_t\right]x_t\\
\frac{n_{t+1}}{n_t} &= \left[R^l_{t+1}-R_{t+1}\right]\phi^l_t+\left[\frac{e_{t+1}}{e_t}(R^{l*}_{t+1}-R^*_{t+1})\right]\phi^{l*}_t\\
&+\left[R^b_{t+1}-R_{t+1}\right]\phi^b_t+\left[R_{t+1}-\frac{e_{t+1}}{e_t}R^*_{t+1}\right]\left(\phi^l_t+\phi^{l*}_t+\phi^b_t\right)x_t+R_{t+1}
\end{aligned}
$$

Thus, bank's problem can be rewritten as the following form:

$$
\begin{aligned}
\psi_t &= \max_{\phi^l_t,\phi^{l*}_t\phi^b_t,x_t} \quad \mu^l_t\phi^l_t+(\mu^{l*}_t+\mu^{d*}_t)\phi^{l*}_t+\mu^b_t\phi^b_t+\mu^{d*}_t\left(\phi^l_t+\phi^{l*}_t+\phi^b_t\right)x_t+\upsilon_t\\
&\quad \text{subject to:}\\
\psi_t &- \Theta(x_t)\left[\phi^l_t+\varpi^*\phi^{l*}_t+\varpi^b\phi^b_t\right]\geq 0
\end{aligned}
$$

where

$$\mu^l_t = \mathbb{E}_t\left[\Omega_{t+1}\left(R^l_{t+1}-R_{t+1}\right)\right] \tag{1}$$

$$\mu^{l*}_t = \mathbb{E}_t\left[\Omega_{t+1}\left(\frac{e_{t+1}}{e_t}R^{l*}_{t+1}-R_{t+1}\right)\right] \tag{2}$$

$$\mu^b_t = \mathbb{E}_t\left[\Omega_{t+1}\left(R^b_{t+1}-R_{t+1}\right)\right] \tag{3}$$

$$\mu^{d*}_t = \mathbb{E}_t\left[\Omega_{t+1}\left(R_{t+1}-\frac{e_{t+1}}{e_t}R^*_{t+1}\right)\right] \tag{4}$$

$$\upsilon_t = \mathbb{E}_t\left[\Omega_{t+1}R_{t+1}\right] \tag{5}$$

$$\Omega_{t+1} = \Lambda_{t,t+1}(1-\sigma+\sigma\psi_{t+1}) \tag{6}$$

We can interpret Ω_{t+1} as the stochastic discount factor of the banker, μ^l_t as the excess return of domestic currency loans over home deposit, μ^l_t is the excess return of foreign currency loans over home deposit, μ^b_t the excess return of sterilized bonds over home deposit, and μ^{d*}_t as the cost advantage of foreign currency debt over home deposit. Note that at the optimal ratios, the following equation will be satisfied:

$$\psi_t = \mu^l_t\phi^l_t+(\mu^{l*}_t+\mu^{d*}_t)\phi^{l*}_t+\mu^b_t\phi^b_t+\mu^{d*}_t\left(\phi^l_t+\phi^{l*}_t+\phi^b_t\right)x_t+\upsilon_t \tag{7}$$

Let λ^b_t be the Lagrange multiplier of the associated incentive restriction, then the problem becomes:

$$
\begin{aligned}
\mathcal{L}_t = \max_{\phi^l_t,\phi^{l*}_t,\phi^b_t,x_t} & \mu^l_t\phi^l_t+(\mu^{l*}_t+\mu^{d*}_t)\phi^{l*}_t+\mu^b_t\phi^b_t+\mu^{d*}_t\left(\phi^l_t+\phi^{l*}_t+\phi^b_t\right)x_t+\upsilon_t\\
&+\lambda^b_t\Big[\mu^l_t\phi^l_t+(\mu^{l*}_t+\mu^{d*}_t)\phi^{l*}_t+\mu^b_t\phi^b_t+\mu^{d*}_t\left(\phi^l_t+\phi^{l*}_t+\phi^b_t\right)x_t+\upsilon_t\\
&-\Theta(x_t)\left(\phi^l_t+\varpi^*\phi^{l*}_t+\varpi^b\phi^b_t\right)\Big]
\end{aligned}
$$

$$\mathcal{L}_t = \max_{\phi_t^l, \phi_t^{l*}, \phi_t^b, x_t} (1+\lambda_t^b)\left[\mu_t^l \phi_t^l + (\mu_t^{l*} + \mu_t^{d*})\phi_t^{l*} + \mu_t^b \phi_t^b + \mu_t^{d*}\left(\phi_t^l + \phi_t^{l*} + \phi_t^b\right) x_t + \upsilon_t\right]$$
$$- \lambda_t^b \Theta(x_t)\left(\phi_t^l + \varpi^* \phi_t^{l*} + \varpi^b \phi_t^b\right)$$

Then, the first order conditions (FOCs) for this problem are:

$$\begin{aligned}
\phi_t^l: \quad & (1+\lambda_t^b)[\mu_t^l + \mu_t^{d*} x_t] - \lambda_t^b \Theta(x_t) = 0 \\
\phi_t^{l*}: \quad & (1+\lambda_t^b)[\mu_t^{l*} + \mu_t^{d*} + \mu_t^{d*} x_t] - \varpi^* \lambda_t^b \Theta(x_t) = 0 \\
\phi_t^b: \quad & (1+\lambda_t^b)[\mu_t^b + \mu_t^{d*} x_t] - \varpi^b \lambda_t^b \Theta(x_t) = 0 \\
x_t: \quad & (1+\lambda_t^b)\mu_t^{d*}\left(\phi_t^l + \phi_t^{l*} + \phi_t^b\right) - \lambda_t^b\left(\phi_t^l + \varpi^* \phi_t^{l*} + \varpi^b \phi_t^b\right)\partial_x \Theta(x_t) = 0 \\
\text{slackness}: \quad & \lambda_t^b\left[\psi_t - \Theta(x_t)\left(\phi_t^l + \varpi^* \phi_t^{l*} + \varpi^b \phi_t^b\right)\right] = 0
\end{aligned}$$

We assume that $\lambda_t^b > 0$ and the incentive constraint is binding. Thus

$$\begin{aligned}
\mu_t^l + \mu_t^{d*} x_t &= \frac{\lambda_t^b}{1+\lambda_t^b}\Theta(x_t) \\
\mu_t^{l*} + \mu_t^{d*}(1+x_t) &= \frac{\lambda_t^b}{1+\lambda_t^b}\varpi^* \Theta(x_t) \\
\mu_t^b + \mu_t^{d*} x_t &= \frac{\lambda_t^b}{1+\lambda_t^b}\varpi^b \Theta(x_t) \\
\mu_t^{d*}\left(\phi_t^l + \phi_t^{l*} + \phi_t^b\right) &= \frac{\lambda_t^b}{1+\lambda_t^b}\left(\phi_t^l + \varpi^* \phi_t^{l*} + \varpi^b \phi_t^b\right)\partial_x \Theta(x_t) \\
\Theta(x_t)\left(\phi_t^l + \varpi^* \phi_t^{l*} + \varpi^b \phi_t^b\right) &= \left(\mu_t^l + \mu_t^{d*} x_t\right)\phi_t^l + \left(\mu_t^{l*} + \mu_t^{d*} x_t\right)\phi_t^{l*} + \left(\mu_t^b + \mu_t^{d*} x_t\right)\phi_t^b + \upsilon_t
\end{aligned}$$

Dividing the first condition by the second and third:

$$\mu_t^{l*} = \varpi^* \mu_t^l - [(1-\varpi^*) x_t + 1]\mu_t^{d*} \tag{8}$$
$$\mu_t^b = \varpi^b \mu_t^l - \left(1 - \varpi^b\right)\mu_t^{d*} x_t \tag{9}$$

Considering the incentive constraint we can rearrange to obtain:

$$\phi_t^l = \Phi_t - \varpi^* \phi_t^{l*} - \varpi^b \phi_t^b \tag{10}$$
$$\Phi_t = \frac{\upsilon_t}{\Theta(x_t) - (\mu_t^l + \mu_t^{d*} x_t)} \tag{11}$$

Note Φ_t defines the maximum weighted leverage ratio induced by the moral hazard problem[22]. We can see that, whenever $\varpi^*, \varpi^b > 0$, private loans and sterilized bonds are substitutes in the portfolio of banks.

Using the fourth optimality condition:

$$\mu_t^{d*}\left(\phi_t^l + \phi_t^{l*} + \phi_t^b\right) = \left(\mu_t^l + \mu_t^{d*} x_t\right)\frac{\partial_x \Theta(x_t)}{\Theta(x_t)}\Phi_t$$

[22]Note that this restriction can be rewritten as:

$$l_t \leq \theta_t n_t$$

where $\theta_t = \Phi_t - \varpi^* \phi_t^{l*} - \varpi^b \phi_t^b$. This type of collateral constraint were popularized by Kiyotaki and Moore (1997) and is used in Chang (2019) to capture foreign debt limits that are faced by the financial system in emerging economies.

$$\mu_t^{d*}\left(\Phi_t + (1-\varpi^*)\,\phi_t^{l*} + \left(1-\varpi^b\right)\phi_t^b\right) = \left(\mu_t^l + \mu_t^{d*}x_t\right)\frac{\partial_x\Theta(x_t)}{\Theta(x_t)}\Phi_t$$
$$\mu_t^{d*}\left(\Phi_t + (1-\varpi^*)\,\phi_t^{l*} + \left(1-\varpi^b\right)\phi_t^b\right) = \left(\mu_t^l + \mu_t^{d*}x_t\right)\frac{\partial_x\Theta(x_t)}{\Theta(x_t)}\Phi_t$$
$$\mu_t^{d*}(1-\varpi^*)\phi_t^{l*} + \mu_t^{d*}(1-\varpi^b)\phi_t^b = \left[\left(\mu_t^l + \mu_t^{d*}x_t\right)\frac{\partial_x\Theta(x_t)}{\Theta(x_t)} - \mu_t^{d*}\right]\Phi_t$$

Hence, the fifth equation for solving bank's problem is[23]:

$$(1-\varpi^*)\phi_t^{l*} + (1-\varpi^b)\phi_t^b = \left[\left(\frac{\mu_t^l}{\mu_t^{d*}} + x_t\right)\frac{\partial_x\Theta(x_t)}{\Theta(x_t)} - 1\right]\Phi_t \tag{12}$$

Financial System Aggregation. We have solved the problem for an individual bank but not for the aggregate banking sector. From eq. (8), we see that the determination of the foreign debt - weighted asset ratio does not depend on bank-specific factors, then this equation is also satisfied at entire banking sector. The same logic applies for eq. (9), eq. (10), eq. (12). Then,

$$\phi_t^l = \frac{L_t}{N_t} \tag{13}$$
$$\phi_t^{l*} = \frac{e_tL_t^*}{N_t} \tag{14}$$
$$\phi_t^b = \frac{B_t}{N_t} \tag{15}$$
$$x_t = \frac{e_tD_t^* - e_tL_t^*}{L_t + e_tL_t^* + B_t} \tag{16}$$

Since the aggregate level of sterilized bonds B_t are determined by the monetary authority and N_t is a state variable, then, in the whole financial system, ϕ_t^b is given. However, now the vector (R_t^l, R_t^{l*}, R_t^b) is not given anymore. The equations which help in the determination of this vector is the law of motion of the aggregated bank's net worth and credit demand functions. The aggregate net worth of banks evolves according to

$$N_{t+1} = \sigma\left(R_{t+1}^lL_t + R_{t+1}^{l*}e_{t+1}L_t^* + R_{t+1}^bB_t - R_{t+1}D_t - e_{t+1}R_{t+1}^*D_t^*\right)$$
$$+ \xi\left(R_{t+1}^lL_t + R_{t+1}^{l*}e_{t+1}L_t^* + R_{t+1}^bB_t\right)$$
$$N_{t+1} = (\sigma+\xi)\left(R_{t+1}^lL_t + R_{t+1}^{l*}e_{t+1}L_t^* + R_{t+1}^bB_t\right) - \sigma R_{t+1}D_t - \sigma e_{t+1}R_{t+1}^*D_t^* \tag{17}$$

Aggregate Currency Mismatch - Case 3. Given x_t and n_t,

$$V_t = \max_{l_t,l_t^*,b_t,d_t,d_t^*} \quad \mathbb{E}_t\left[\Lambda_{t,t+1}\left\{(1-\sigma)n_{t+1} + \sigma V_{t+1}\right\}\right]$$

subject to:

$$l_t + e_tl_t^* + b_t = n_t + d_t + e_td_t^*$$
$$n_{t+1} = R_{t+1}^ll_t + R_{t+1}^{l*}e_{t+1}l_t^* + R_{t+1}^bb_t - R_{t+1}d_t - e_{t+1}R_{t+1}^*d_t^*$$

[23] Note that if $1 = \varpi^*$ and $1 = \varpi^b$, we arrive to the a similar solution of Aoki et al. (2018) :

$$1 = \left(\frac{\mu_t^l}{\mu_t^{d*}} + x_t\right)\frac{\partial_x\Theta(x_t)}{\Theta(x_t)}$$

If $\varpi^* = \varpi^b = 1$, we get the same solution of Aoki et al. (2018) for the whole financial system since returns are the same across different types of assets.

$$V_t \geq \Theta(x_t)\left[l_t + \varpi^* e_t l_t^* + \varpi^b b_t\right]$$

Let $\psi_t = \frac{V_t}{n_t}$, $\phi_t^l = \frac{l_t}{n_t}$, $\phi_t^l = \frac{e_t l_t^*}{n_t}$, and $\phi_t^b = \frac{b_t}{n_t}$, then the objective function can be rewritten as

$$\psi_t = \mathbb{E}_t\left[\Lambda_{t,t+1}(1-\sigma+\sigma\psi_{t+1})\frac{n_{t+1}}{n_t}\right]$$

Moreover, let $\phi_t^{d*} = \frac{e_t d_t^*}{n_t}$

$$\begin{aligned}
\frac{n_{t+1}}{n_t} &= R_{t+1}^l \phi_t^l + R_{t+1}^{l*}\frac{e_{t+1}}{e_t}\phi_t^* + R_{t+1}^b\phi_t^b - R_{t+1}\frac{d_t}{n_t} - R_{t+1}^*\frac{e_{t+1}}{e_t}\phi_t^{d*} \\
\frac{n_{t+1}}{n_t} &= R_{t+1}^l \phi_t^l + R_{t+1}^{l*}\frac{e_{t+1}}{e_t}\phi_t^* + R_{t+1}^b\phi_t^b - R_{t+1}\left[\phi_t^l + \phi_t^{l*} + \phi_t^b - 1 - \phi_t^{d*}\right] - R_{t+1}^*\frac{e_{t+1}}{e_t}\phi_t^{d*} \\
\frac{n_{t+1}}{n_t} &= [R_{t+1}^l - R_{t+1}]\phi_t^l + \left[R_{t+1}^{l*}\frac{e_{t+1}}{e_t} - R_{t+1}\right]\phi_t^* + [R_{t+1}^b - R_{t+1}]\phi_t^b \\
&\quad + \left[R_{t+1} - R_{t+1}^*\frac{e_{t+1}}{e_t}\right]\phi_t^{d*} + R_{t+1}
\end{aligned}$$

Then, the bank's problem can be rewritten as

$$\begin{aligned}
\psi_t = &\max_{\phi_t^l,\phi_t^{l*}\phi_t^b,\phi_t^{d*}} \quad \mu_t^l\phi_t^l + \mu_t^{l*}\phi_t^{l*} + \mu_t^b\phi_t^b + \mu_t^{d*}\phi_t^{d*} + v_t \\
&\text{subject to:} \\
\psi_t &- \Theta(x_t)\left[\phi_t^l + \varpi^*\phi_t^{l*} + \varpi^b\phi_t^b\right] \geq 0
\end{aligned}$$

FOCs

$$\begin{aligned}
\phi_t^l: &\quad (1+\lambda_t^b)\mu_t^l - \lambda_t^b\Theta(x_t) = 0 \\
\phi_t^{l*}: &\quad (1+\lambda_t^b)\mu_t^{l*} - \varpi^*\lambda_t^b\Theta(x_t) = 0 \\
\phi_t^b: &\quad (1+\lambda_t^b)\mu_t^b - \varpi^b\lambda_t^b\Theta(x_t) = 0 \\
\phi_t^{d*}: &\quad (1+\lambda_t^b)\mu_t^{d*} = 0 \\
\text{slackness}: &\quad \lambda_t^b\left[\psi_t - \Theta(x_t)\left(\phi_t^l + \varpi^*\phi_t^{l*} + \varpi^b\phi_t^b\right)\right] = 0
\end{aligned}$$

Rearranging

$$\begin{aligned}
\mu_t^{d*} &= 0 \\
\mu_t^{l*} &= \varpi^*\mu^l \\
\mu_t^b &= \varpi^b\mu^l
\end{aligned}$$

thus

$$\psi = \mu^l\Phi_t + v_t \tag{18}$$

Hence,

$$\Phi_t = \frac{v_t}{\Theta(x_t) - \mu^l} \tag{19}$$

C.2 Solving Worker's Problem

Objective Function:

$$U_t = (1-\beta)\mathbb{E}_t\left[\sum_{j=0}^{\infty}\beta^j \frac{1}{1-\gamma}\left(C_{t+j} - \mathcal{H}C_{t+j-1} - \frac{\zeta_0}{1+\zeta}H_{t+j}^{1+\zeta}\right)^{1-\gamma}\right]$$

Budget Restriction:

$$\begin{aligned}
&C_t + D_t + B_t^g + e_t\left[D_t^{*,h} + \frac{\kappa_{D*}}{2}\left(D_t^{*,h} - \overline{D}^{*,h}\right)^2\right] + \left[\mathcal{S}_t + \frac{\kappa_S}{2}\left(\mathcal{S}_t - \overline{\mathcal{S}}\right)^2\right] + T_t \\
&\quad = w_t H_t + \Pi_t + R_t D_{t-1} + R_t^* e_t D_{t-1}^{*,h} + R_t^{knc}\mathcal{S}_{t-1}
\end{aligned}$$

First Order Conditions:

$$\mathbb{E}_t u_{ct} w_t = \zeta_0 H_t^{\zeta}\left(C_t - \mathcal{H}C_{t-1} - \frac{\zeta_0}{1+\zeta}H_t^{1+\zeta}\right)^{-\gamma} \tag{1}$$

$$1 = \mathbb{E}_t\left[R_{t+1}\Lambda_{t,t+1}\right] \tag{2}$$

$$D_t^{*,h} = \overline{D}^{*,h} + \frac{\mathbb{E}_t\left[\Lambda_{t,t+1}\left(\frac{e_{t+1}}{e_t}R_{t+1}^* - R_{t+1}\right)\right]}{\kappa_{D*}} \tag{3}$$

$$\mathcal{S}_t = \overline{\mathcal{S}} + \frac{\mathbb{E}_t\left[\Lambda_{t,t+1}\left(R_{t+1}^S - R_{t+1}\right)\right]}{\kappa_S} \tag{4}$$

with

$$u_{ct} = \left(C_t - \mathcal{H}C_{t-1} - \frac{\zeta_0}{1+\zeta}H_t^{1+\zeta}\right)^{-\gamma} - \mathcal{H}\beta\mathbb{E}_t\left(C_{t+1} - \mathcal{H}C_t - \frac{\zeta_0}{1+\zeta}H_{t+1}^{1+\zeta}\right)^{-\gamma} \tag{5}$$

$$\Lambda_{t,t+1} = \beta\frac{u_{c,t+1}}{u_{ct}} \tag{6}$$

C.3 Price Setting

Given $p_{j,t-1}^{nc}$, $k_{j,t-1}^{nc}$, $\mathcal{S}_{j,t-1}$, and $\mathcal{F}_{j,t-1}$, a representative intermediate good producer chooses $\{h_{j,t+s}, m_{j,t+s}, p_{j,t+s}^{nc}, y_{j,t+s}^{nc}, k_{j,t+s}^{nc}, \mathcal{S}_{j,t+s}, \mathcal{F}_{j,t+s}\}_{s\geq 0}$ to maximize

$$\begin{aligned}
\max\ \mathbb{E}_t\Big[\sum_{s=0}^{\infty}\Lambda_{t,t+s}\Big\{&\frac{p_{j,t+s}^{nc}}{P_{t+s}^{nc}}y_{j,t+s}^{nc} - w_{t+s}h_{j,t+s} - e_{t+s}m_{j,t+s} - \Theta_{t+s}\left(\frac{p_{t+s}^{nc}}{p_{t+s-1}^{nc}}\right) \\
&+ q_{t+s}^{nc}\lambda_{nc}k_{j,t+s-1}^{nc} - R_{t+s}^S\mathcal{S}_{j,t+s-1} - R_{t+s}^F\mathcal{F}_{j,t+s-1}\Big\}\Big]
\end{aligned}$$

subject to:

$$\begin{aligned}
0 &= y_{j,t}^{nc} - A_t^{nc}\left(\frac{k_{j,t-1}^{nc}}{\alpha_k}\right)^{\alpha_k}\left(\frac{m_{j,t}}{\alpha_m}\right)^{\alpha_m}\left(\frac{h_{j,t}}{1-\alpha_k-\alpha_m}\right)^{1-\alpha_k-\alpha_m} \\
0 &= y_{j,t}^{nc} - \left(\frac{p_{j,t}^{nc}}{P_t^{nc}}\right)^{-\eta}Y_t^{nc} \\
0 &= \mathcal{S}_{j,t} + \mathcal{F}_{j,t} - q_t^{nc}k_{j,t}^{nc}
\end{aligned}$$

Denoting the Lagrangian multipliers: mc_t, $\mathcal{L}_{1t}$, and $\mathcal{L}_{2t}$ respectively, and let define

$$z_t = mc_t A_t (k_{jt-1}^{nc})^{\alpha_k - 1}\left(\frac{m_{jt}}{\alpha_m}\right)^{\alpha_m}\left(\frac{h_{jt}}{1-\alpha_k-\alpha_m}\right)^{1-\alpha_k-\alpha_m}$$

$$R_t^{knc} = \frac{\lambda_{nc} q_t^{nc} + z_t}{q_{t-1}^{nc}}$$

The necessary conditions are:

$$\begin{aligned}
h_{j,t}: &\quad 0 = -w_t + mc_t A_t^{nc} \left(\frac{k_{j,t-1}^{nc}}{\alpha_k}\right)^{\alpha_k} \left(\frac{m_{j,t}}{\alpha_m}\right)^{\alpha_m} h_{j,t}^{-\alpha_k-\alpha_m} \\
m_{jt}: &\quad 0 = -e_t + mc_t A_t^{nc} m_{j,t}^{\alpha_m-1} \left(\frac{k_{j,t-1}^{nc}}{\alpha_k}\right)^{\alpha_k} \left(\frac{h_{j,t}}{1-\alpha_k-\alpha_m}\right)^{1-\alpha_k-\alpha_m} \\
p_{j,t}^{nc}: &\quad 0 = \frac{1}{P_t^{nc}} y_{jt}^{nc} - \frac{1}{p_{j,t-1}^{nc}} \Theta_t' - \mathcal{L}_{1t} \eta (p_{j,t}^{nc})^{-\eta-1} \left(\frac{1}{P_t^{nc}}\right)^{-\eta} Y_t^{nc} + \mathbb{E}_t \left[\Lambda_{t,t+1} \frac{p_{j,t+1}^{nc}}{(p_{j,t}^{nc})^2} \Theta_{t+1}'\right] \\
y_{j,t}^{nc}: &\quad 0 = \frac{p_{j,t}^{nc}}{P_t^{nc}} - mc_t - \mathcal{L}_{1t} \\
k_{j,t}^{nc}: &\quad 0 = -\mathcal{L}_{2t} q_t^{nc} + \mathbb{E}_t \Lambda_{t,t+1} \left[\lambda_{nc} q_{t+1}^{nc} + z_{t+1}\right] \\
\mathcal{S}_{j,t}: &\quad 0 = \mathcal{L}_{2,t} - \mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^S \\
\mathcal{F}_{j,t}: &\quad 0 = \mathcal{L}_{2,t} - \mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^F
\end{aligned}$$

along with the three restrictions written above. We can rearrange and aggregate to get the following **optimal conditions**:

$$Y_t^{nc} = A_t^{nc} \left(\frac{K_{t-1}^{nc}}{\alpha_k}\right)^{\alpha_k} \left(\frac{M_t}{\alpha_m}\right)^{\alpha_m} \left(\frac{H_t}{1-\alpha_k-\alpha_m}\right)^{1-\alpha_k-\alpha_m} \tag{7}$$

$$z_t = \alpha_k mc_t \frac{Y_t^{nc}}{K_{t-1}^{nc}} \tag{8}$$

$$e_t = \alpha_m mc_t \frac{Y_t^{nc}}{M_t} \tag{9}$$

$$mc_t = \frac{1}{A_t^{nc}} z_t^{\alpha_k} e_t^{\alpha_m} w_t^{1-\alpha_k-\alpha_m} \tag{10}$$

$$q_t^{nc} K_t^{nc} = \mathcal{S}_t + \mathcal{F}_t \tag{11}$$

$$R_t^{knc} = \frac{\lambda_{nc} q_t^{nc} + z_t}{q_{t-1}^{nc}} \tag{12}$$

$$0 = \mathbb{E}_t \left[\Lambda_{t,t+1} \left(R_{t+1}^{knc} - R_{t+1}^S\right)\right] \tag{13}$$

$$0 = \mathbb{E}_t \left[\Lambda_{t,t+1} \left(R_{t+1}^{knc} - R_{t+1}^F\right)\right] \tag{14}$$

Moreover, regarding the optimal pricing

$$\begin{aligned}
&\frac{1}{P_t^{nc}} \left(\frac{p_{j,t}^{nc}}{P_t^{nc}}\right)^{-\eta} Y_t^{nc} - \frac{\eta}{p_{j,t}^{nc}} \left(\frac{p_{j,t}^{nc}}{P_t^{nc}} - mc_t\right) \left(\frac{p_{j,t}^{nc}}{P_t^{nc}}\right)^{-\eta} Y_t^{nc} \\
&\quad - \frac{\kappa}{p_{j,t-1}^{nc}} \left(\frac{p_{j,t}^{nc}}{p_{j,t-1}^{nc}} - 1\right) Y_t^{nc} + \kappa \mathbb{E}_t \left[\Lambda_{t,t+1} \frac{p_{j,t+1}^{nc}}{(p_{j,t}^{nc})^2} \left(\frac{p_{j,t+1}^{nc}}{p_{j,t}^{nc}} - 1\right) Y_{t+1}^{nc}\right] = 0
\end{aligned}$$

Considering the symmetric equilibrium $p_{j,t}^{nc} = P_t^{nc}$ for all $j \in [0,1]$ and denoting $\pi_t = \frac{P_t^{nc}}{P_{t-1}^{nc}} - 1$, then

$$\begin{aligned}
0 = &\frac{1}{P_t^{nc}} Y_t^{nc} - \frac{\eta}{P_t^{nc}} (1 - mc_t) Y_t^{nc} - \frac{\kappa}{P_{t-1}^{nc}} \left(\frac{P_t^{nc}}{P_{t-1}^{nc}} - 1\right) Y_t^{nc} \\
&+ \kappa \mathbb{E}_t \left[\Lambda_{t,t+1} \frac{P_{t+1}^{nc}}{(P_t^{nc})^2} \left(\frac{P_{t+1}^{nc}}{P_t^{nc}} - 1\right) Y_{t+1}^{nc}\right]
\end{aligned}$$

$$
\begin{aligned}
0 &= Y_t^{nc} - \eta\left(1 - mc_t\right) Y_t^{nc} - \kappa(1+\pi_t)\pi_t Y_t^{nc} + \kappa \mathbb{E}_t\left[\Lambda_{t,t+1}(1+\pi_{t+1})\pi_{t+1} Y_{t+1}^{nc}\right] \\
0 &= 1 - \eta\left(1 - mc_t\right) - \kappa(1+\pi_t)\pi_t + \kappa \mathbb{E}_t\left[\Lambda_{t,t+1}(1+\pi_{t+1})\pi_{t+1} \frac{Y_{t+1}^{nc}}{Y_t^{nc}}\right]
\end{aligned}
$$

Hence, we obtain the **Phillips Curve equation**:

$$
(1+\pi_t)\pi_t = \frac{1}{\kappa}(1 - \eta + \eta mc_t) + \mathbb{E}_t\left[\Lambda_{t,t+1}(1+\pi_{t+1})\pi_{t+1} \frac{Y_{t+1}^{nc}}{Y_t^{nc}}\right] \tag{15}
$$

Intermediate good producers also need to decide the optimal composition for $\mathcal{F}_t$. First note that:

$$
R_t^F \mathcal{F}_{j,t-1} = R_t^l l_{j,t-1} + R_t^{l*} e_t l_{j,t-1}^* \tag{16}
$$

Then, the composition problem is:

$$
\min_{l_{j,t}, l_{j,t}^*} \quad \mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^F \mathcal{F}_{j,t} = \mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^l l_{j,t} + \mathbb{E}_t \Lambda_{t,t+1} e_{t+1} R_{t+1}^{l*} l_{j,t}^*
$$

subject to:

$$
\mathcal{F}(l_{j,t}, e_t l_{j,t}^*) \leq \mathcal{F}_{j,t}
$$

Let $\mathcal{L}_3$ be the Lagrangian multiplier associated with the restriction, then the **optimal conditions** are:

$$
\begin{aligned}
0 &= \mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^l - \mathcal{L}_3 \mathcal{F}_1(l_{j,t}, e_t l_{j,t}^*) \\
0 &= \mathbb{E}_t \Lambda_{t,t+1} \frac{e_{t+1}}{e_t} R_{t+1}^{l*} - \mathcal{L}_3 \mathcal{F}_2(l_{j,t}, e_t l_{j,t}^*)
\end{aligned}
$$

Since we assume that $\mathcal{F}()$ is an homogeneous function, then $\mathcal{L}_3 = \mathbb{E}_t \Lambda_{t,t++1} R_{t+1}^F$ or equivalently

$$
\begin{aligned}
\mathcal{F}_1(l_{j,t}, e_t l_{j,t}^*) &= \frac{\mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^l}{\mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^F} \\
\mathcal{F}_2(l_{j,t}, e_t l_{j,t}^*) &= \frac{\mathbb{E}_t \Lambda_{t,t+1} \frac{e_{t+1}}{e_t} R_{t+1}^{l*}}{\mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^F}
\end{aligned}
$$

In our baseline parametrization we use the next CES function

$$
\mathcal{F}(l, el^*) = A^e L_t^{1-\delta^f} \left(e_t L_t^*\right)^{\delta^f} \tag{17}
$$

Hence,

$$
l_{j,t} = (1 - \delta^f) \left(\frac{\mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^F}{\mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^l} \right) \mathcal{F}_{j,t} \tag{18}
$$

$$
e_t l_{jt}^* = \delta^f \left(\frac{\mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^F}{\mathbb{E}_t \Lambda_{t,t+1} \frac{e_{t+1}}{e_t} R_{t+1}^{l*}} \right) \mathcal{F}_{j,t} \tag{19}
$$

We finally impose that $\mathcal{S}_t$ is equity so that $R_t^S = R_t^{knc}$.